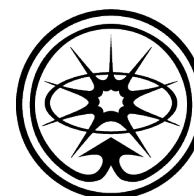


Applications of the chiral superfluidity to QCD

Tigran Kalaydzhyan

ArXiv: 1203.4259, 1102.4334, 1208.0012, 1212.3168,
1111.6733, 1301.6558, 1302.6458, 1302.6510, 1401.5974
+ works in progress



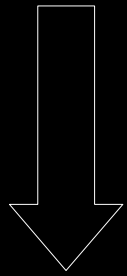
February 16, 2014. Simons Center for Geometry and Physics, NY, USA.

Overview

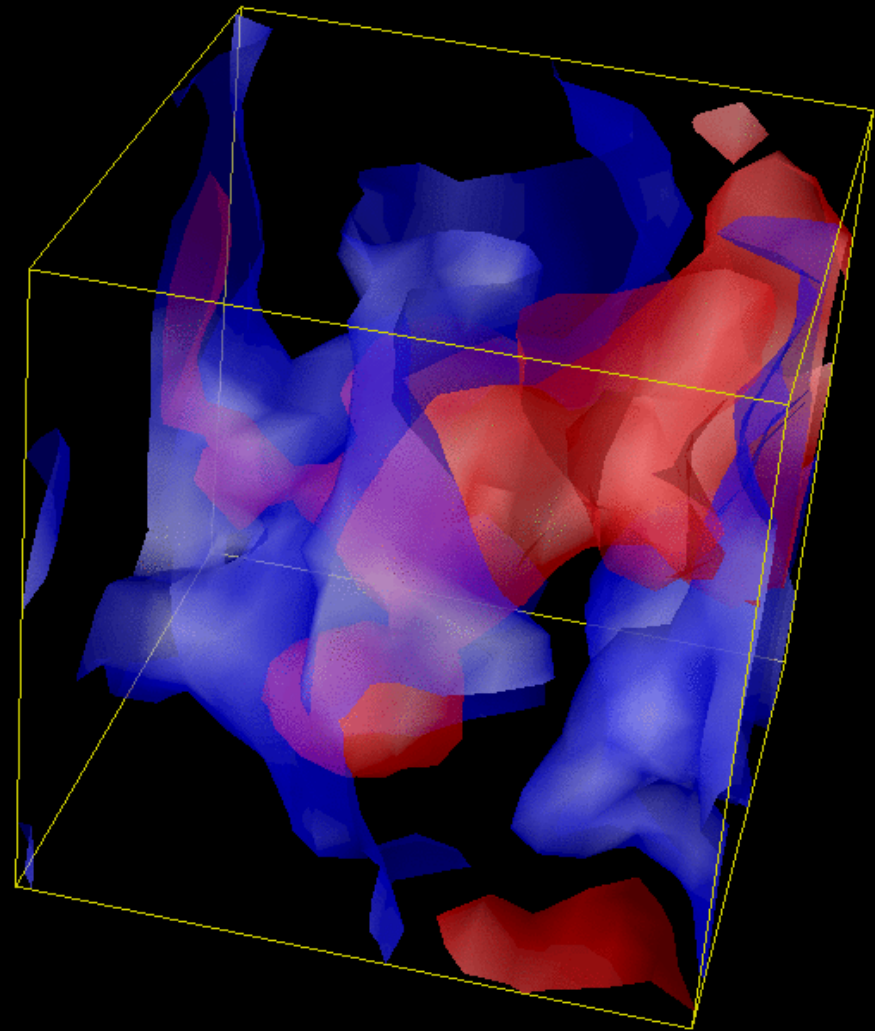
- Motivation. QCD and heavy-ion collisions.
- Transport coefficients.
- Low temperatures, chiral theory.
- High temperature, kinetic theory.
- Intermediate temperatures, sQGP.
- On the role of defects in hydro and QCD.
- Conclusions.

QCD vacuum

$$G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



$$\rho_R \neq \rho_L$$

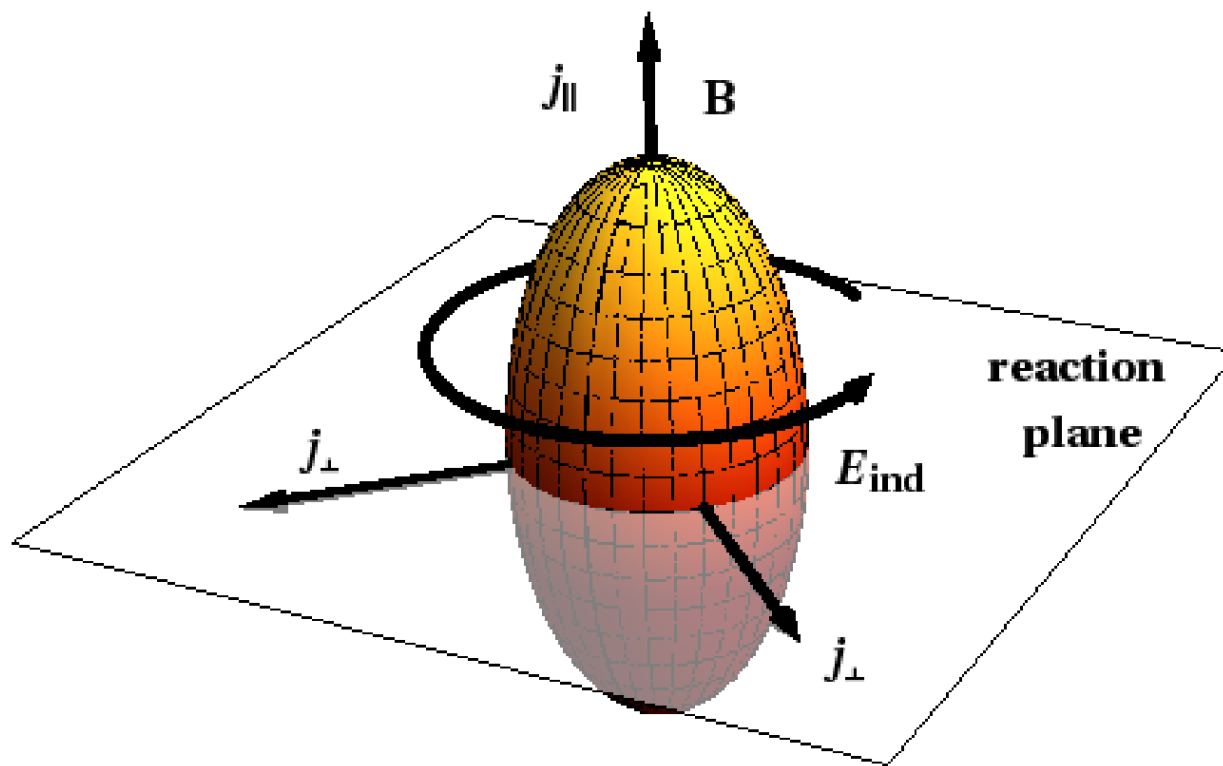


Positive topological
charge density

Negative topological
charge density

For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

Heavy-ion collisions



Excess of positive charge

$$J^{\mu} = C \mu_5 B^{\mu}$$

Chiral Magnetic Effect

$$J^{\mu} = 2 C \mu \mu_5 \omega^{\mu}$$

Chiral Vortical Effect

Excess of negative charge

Fukushima, Kharzeev, McLerran, Warringa (2007)

Vilenkin (1980), Kharzeev, Zhitnitsky (2007), Kharzeev, Son (2011) ...

Anomalous effects

Hydrodynamic equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda + F_5^{\nu\lambda} j_{5\lambda},$$

$$\partial_\mu j_5^\mu = C E^\lambda \cdot B_\lambda + \frac{C}{3} E_5^\lambda \cdot B_{5\lambda},$$

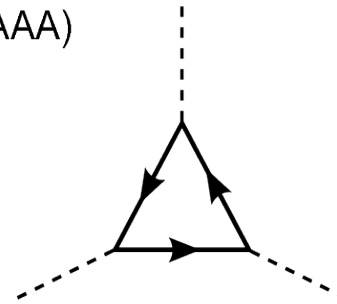
$$\partial_\mu j^\mu = 0$$

where vector and axial currents are

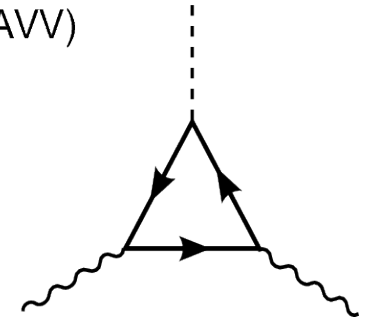
CVE	$\kappa_\omega = 2C\mu\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P} \left[1 + \frac{\mu_5^2}{3\mu^2} \right] \right),$	$\kappa_B = C\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + P} \right),$	CME
AVE	$\xi_\omega = C\mu^2 \left(1 - 2 \frac{\mu_5\rho_5}{\epsilon + P} \left[1 + \frac{\mu_5^2}{3\mu^2} \right] \right),$	$\xi_B = C\mu \left(1 - \frac{\mu_5\rho_5}{\epsilon + P} \right),$	CSE

Anomalies:

(AAA)



(AVV)



$$j^\mu = \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu + \dots$$

$$j_5^\mu = \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \dots$$

Low

temperatures

Cold pions

Gauged WZW action

$$\begin{aligned}
 S = & \frac{f_\pi^2}{4} \int d^4x \operatorname{Tr} [D_\alpha U^\dagger D^\alpha U] \\
 & - \frac{iN_c}{240\pi^2} \int d^5x \epsilon^{\alpha\beta\gamma\delta\zeta} \operatorname{Tr} [R_\alpha R_\beta R_\gamma R_\delta R_\zeta] \\
 & - \frac{N_c}{48\pi^2} \int d^4x \epsilon^{\alpha\beta\gamma\delta} A_\alpha \operatorname{Tr} [Q(L_\beta L_\gamma L_\delta + R_\beta R_\gamma R_\delta)] \\
 & + \frac{iN_c}{24\pi^2} \int d^4x \tilde{F}^{\alpha\beta} A_\alpha \operatorname{Tr} [Q^2(L_\beta + R_\beta) + \frac{1}{2}(QUQU^\dagger L_\beta + QU^\dagger QU R_\beta)]
 \end{aligned}$$

$D_\alpha \equiv \partial_\alpha + i A_\alpha [Q, \cdot]$
 $U = \exp \left(\frac{i}{f_\pi} \pi^a \tau^a \right)$
 $L_\alpha \equiv \partial_\alpha U U^\dagger$
 $R_\alpha \equiv U^\dagger \partial_\alpha U$

Anomaly: $\partial_\alpha j_5^\alpha = -\frac{N_c}{4\pi^2} F_{\alpha\beta} \tilde{F}^{\alpha\beta} \operatorname{Tr} [Q^2 Q_5], \quad Q_5 = \tau^3/2 \text{ or } 1/3$

Cold pions

Gauged WZW action

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Let us study the π^0 condensate. Then, naively, we have the currents

$$j_5^\alpha = f_\pi \partial^\alpha \pi^3 = \rho_5 u_S^\alpha \qquad j^\alpha = -\frac{N_c}{12\pi^2} \mu_5 \tilde{F}^{\alpha\beta} u_\beta^S \qquad j_{5B}^\alpha = 0$$

Cold pions

In the presence of rotation we get a nontrivial topology, since the condensate is, in general, curl-free (the condensate velocity is a gradient of a field).

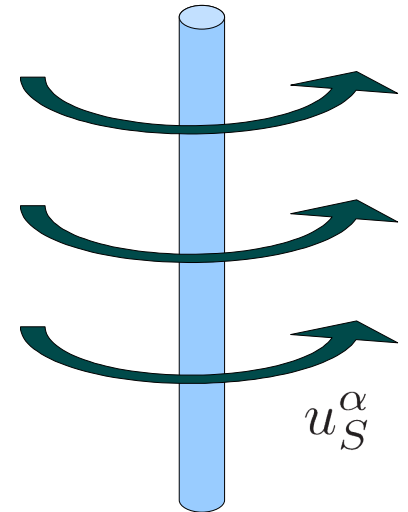
$$[\partial_\alpha^\perp, \partial_\beta^\perp] \pi^a = 2\pi f_\pi \delta^{(2)}(\vec{x}_\perp)$$

This modifies the Maurer–Cartan equations, e.g.

$$L_{[\alpha} L_{\beta]} = \partial_{[\alpha} L_{\beta]} + \sum_{i,a} i\pi \delta(x_i^\alpha) \delta(x_i^\beta) \tau^a$$

the bulk currents

$$j_{5B}^\alpha = \frac{N_c}{72\pi^2 f_\pi^2} \epsilon^{\alpha\beta\gamma\delta} \partial_\beta \pi^3 \partial_\gamma \partial_\delta \pi^3 = \frac{N_c}{36\pi^2} \mu_5^2 \omega_S^\alpha$$



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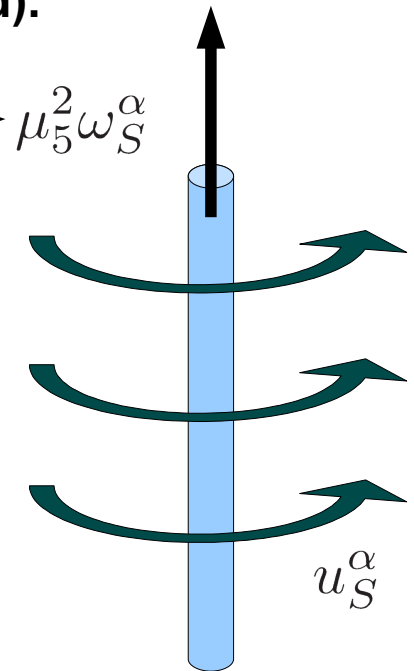
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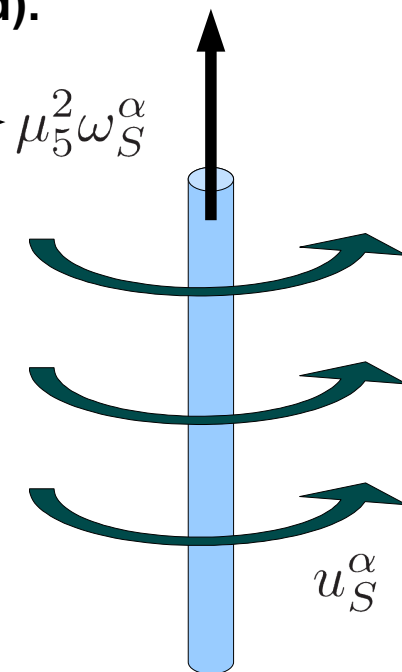
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... and induces a vector current along the vortex (string)

$$j^{\alpha,a} = \frac{N_c \epsilon^{\alpha\beta}}{12\pi f_\pi} (\partial_\beta \pi^b \text{Tr}[Q \tau^b \tau^a] - 2f_\pi A_\beta \text{Tr}[Q \tau^a]) \quad \xrightarrow{\text{only } \pi^0} \quad j^z = -\frac{N_c \mu_5}{36\pi}$$



Cold pions

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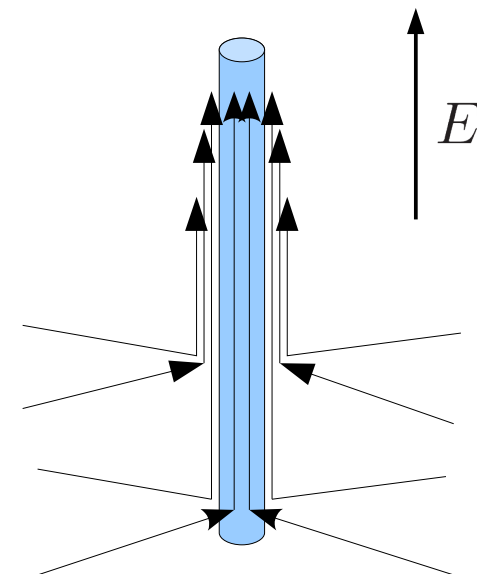
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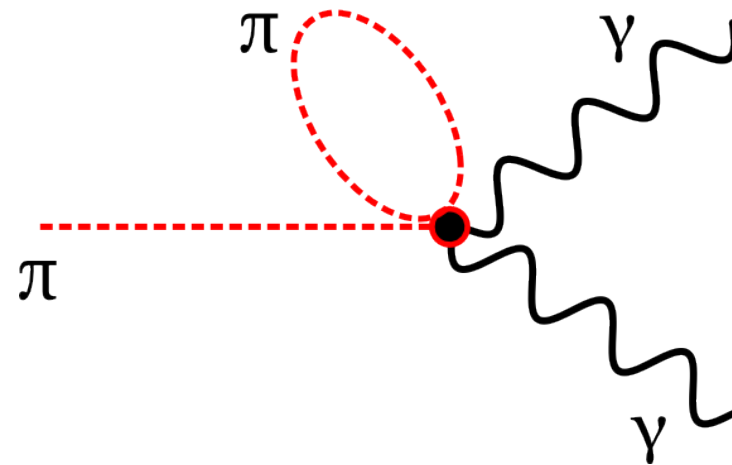
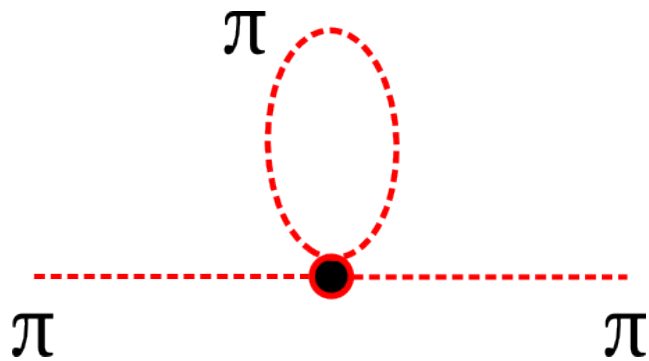
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anomaly inflow:
$$\partial_\alpha j_{\text{bulk}}^\alpha = -\frac{N_c}{12\pi^2 f_\pi} \tilde{F}^{\alpha\beta} \partial_\alpha \partial_\beta \pi^3 \propto E \delta^{(2)}(\vec{x}_\perp)$$



Temperature dependence

Temperature dependence can be obtained from the tadpole resummation.
The pions are excited thermally with the Bose-Einstein distribution



$$\langle \pi^2 \rangle_T = \int \frac{2\pi \delta(p^2)}{e^{\omega/T} - 1} d^4p = \frac{T^2}{12}$$

Renormalized currents:

$$j^\alpha(T) = -\frac{N_c}{12\pi^2} \mu_5 \tilde{F}^{\alpha\beta} u_\beta^S$$

$$j_{5B}^\alpha(T) = \frac{N_c}{36\pi^2} \left(\mu_5^2 - \frac{\mu_5^2}{9f_\pi^2} T^2 \right) \omega_S^\alpha$$

High temperatures

The fraction of condensed phase becomes smaller, vanishing above the critical temperature. The total vorticity is transferred to the normal phase.

$$\Omega = \sum_{s=\pm} \int \frac{d^3p}{(2\pi)^3} \left[\omega_{p,s} + T \sum_{\pm} \log \left(1 + e^{-\frac{\omega_{p,s} \pm \mu}{T}} \right) \right]$$

where $\omega_{p,s}^2 = (p + s\mu_5)^2 + m^2$ Fukushima, Kharzeev, Warringa (2008)

$$j^\alpha = \rho u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu \partial \mu_5} \omega^\alpha + \frac{1}{4} \frac{\partial^3 \Omega}{\partial \mu^2 \partial \mu_5} B^\alpha = \rho u^\alpha + 2C \mu \mu_5 \omega^\alpha + C \mu_5 B^\alpha$$

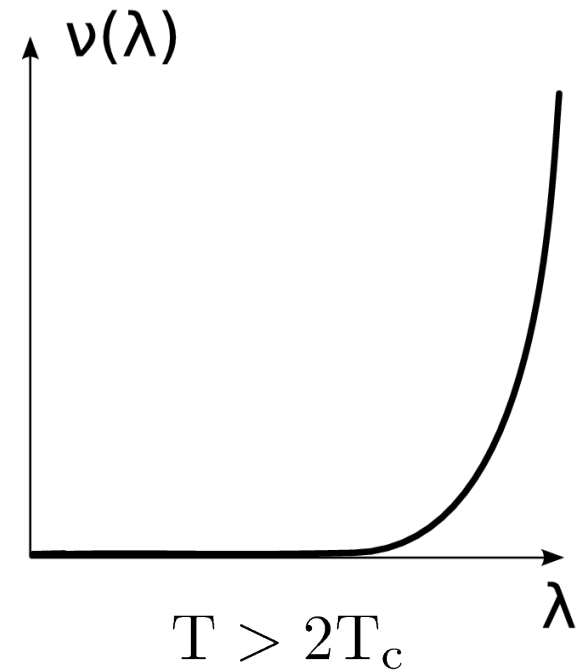
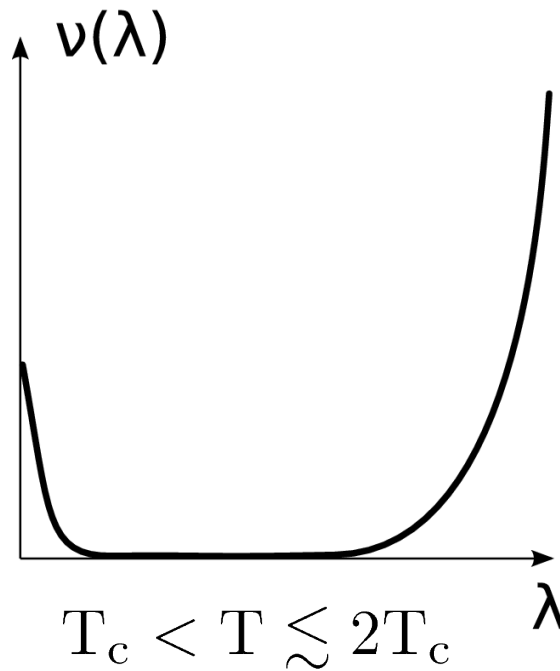
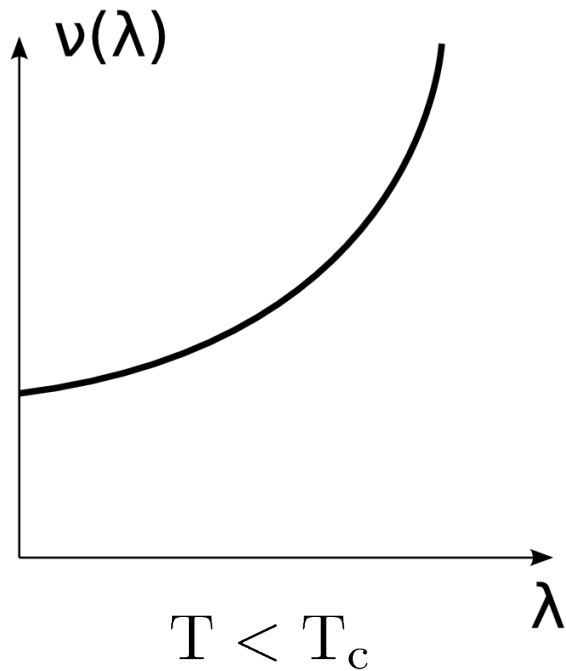
$$\begin{aligned} j_{5B}^\alpha &= \rho_{5B} u^\alpha + \frac{1}{2} \frac{\partial^2 \Omega}{\partial \mu^2} \omega^\alpha + \frac{1}{12} \frac{\partial^3 \Omega}{\partial \mu^3} B^\alpha = \\ &= \rho_{5B} u^\alpha + \left[\frac{1}{2\pi^2} (\mu^2 + \mu_5^2) + \frac{T^2}{6} \right] \omega^\alpha + \frac{\mu}{6\pi^2} B^\alpha \end{aligned}$$

**Intermediate
temperatures**

Insight from the lattice

- Spectrum of the Dirac operator

$$\hat{D}\psi_\lambda = \lambda\psi_\lambda$$

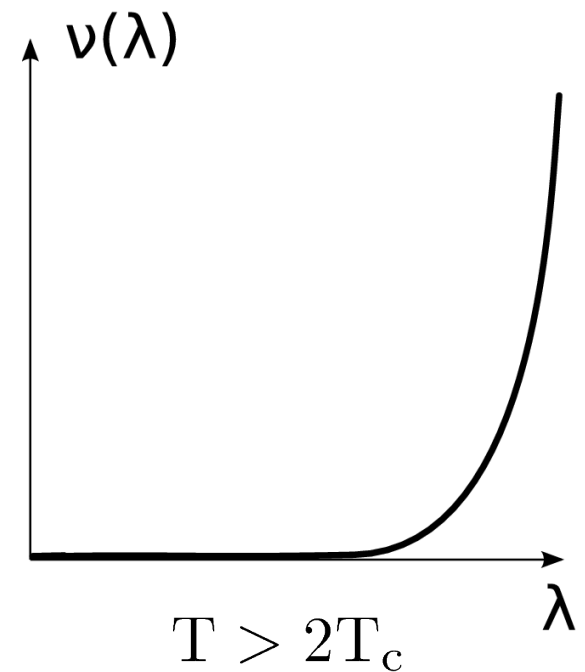
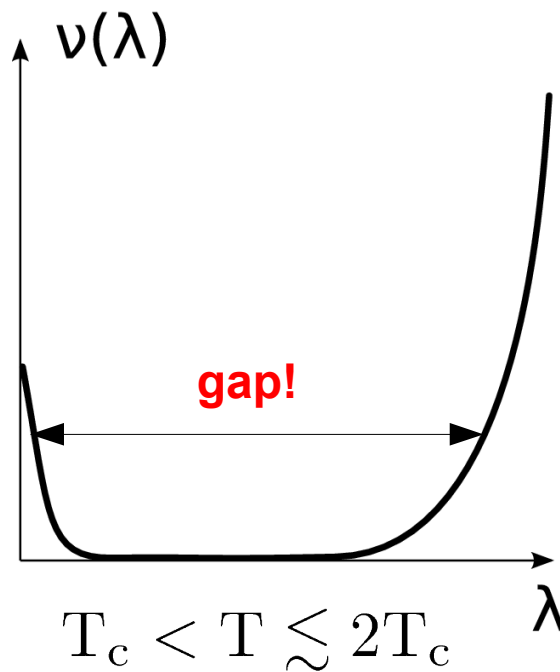
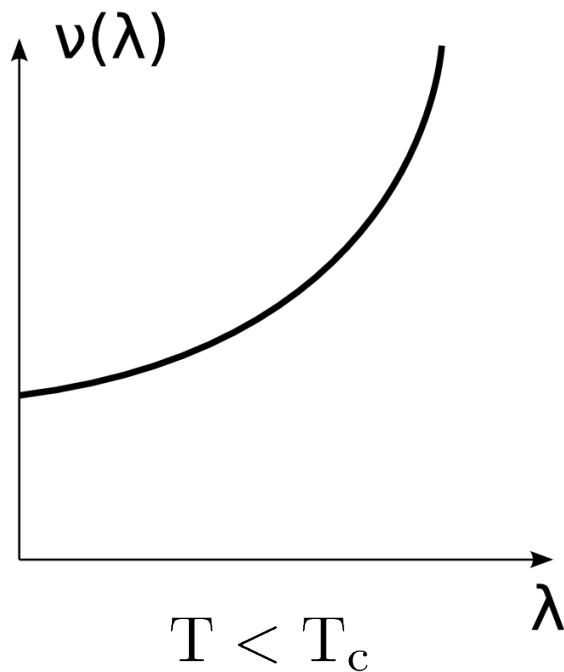


- Chiral properties are described by near-zero modes

Insight from the lattice

- Spectrum of the Dirac operator

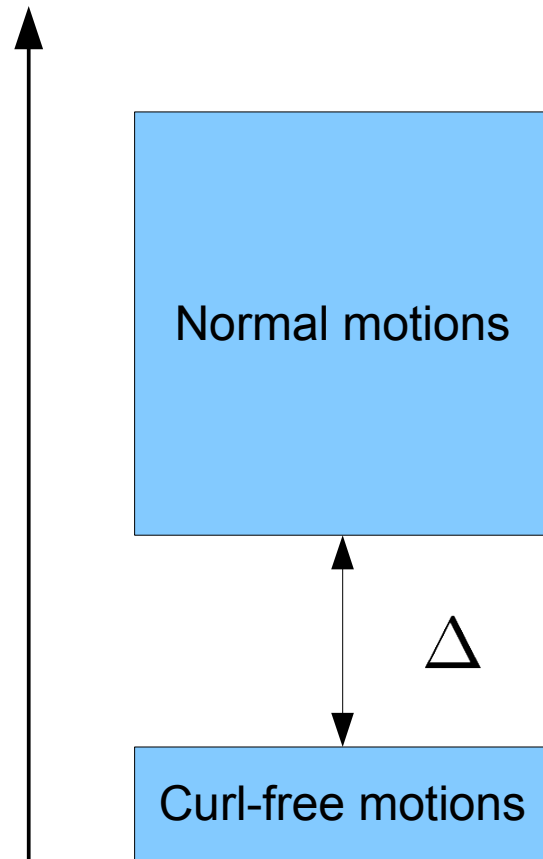
$$\hat{D}\psi_\lambda = \lambda\psi_\lambda$$



- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Why "superfluidity" ?

Energy



AUGUST 15, 1941

PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU

Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ .

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

4D "Bosonization"

The **total effective Euclidean Lagrangian** for QCD×QED reads as

$$\begin{aligned}\mathcal{L}_E^{(4)} = & \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu \\ & + \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \tilde{F}_{\mu\nu} \\ & + \frac{N_c}{24\pi^2} \theta \square^2 \theta - \frac{N_c}{12\pi^2} (\partial^\mu \theta \partial_\mu \theta)^2\end{aligned}$$

Here θ is a result of a gauge-invariant bosonization of the low-lying fermionic modes with finite cutoff Λ and gauged U(1) axial symmetry. The transformation parameter becomes a dynamical axion-like field. The cutoff has a physical meaning.

$$\Lambda_T = \pi \sqrt{\frac{2}{3}} \sqrt{T^2 + \frac{\mu^2}{\pi^2}}$$

$$\Lambda_B = 2\sqrt{|eB|}$$

$$\Lambda_{latt} \simeq 3 \text{ GeV}$$

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

Energy-momentum tensor $\rightarrow \partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda,$

$\partial_\mu J^\mu = 0,$

Axial current $\rightarrow \partial_\mu J_5^\mu = C E^\mu B_\mu,$

4-velocity of the normal component $\rightarrow u^\mu \partial_\mu \theta + \mu_5 = 0,$

Total electric current

Electromagnetic fields

Chiral anomaly coefficient

Josephson equation, defining The axial chemical potential through the bosonized low-lying modes.

Similar to the superfluid dynamics!

Constitutive relations

Solving hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

The diagram illustrates the constitutive relations for the energy-momentum tensor $T^{\mu\nu}$, the current J^μ , and the charge density J_5^μ . Arrows indicate the physical quantities associated with each term in the equations.

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu},$$

Energy density ϵ and Pressure P are associated with the first two terms of $T^{\mu\nu}$. The term $f^2 \partial^\mu \theta \partial^\nu \theta$ is associated with the decay constant f . The term $\tau^{\mu\nu}$ represents dissipative corrections (viscosity, resistance, etc.).

$$J^\mu = \rho u^\mu + C \tilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu,$$

Charge density ρ is associated with the first term of J^μ . The term $C \tilde{F}^{\mu\kappa} \partial_\kappa \theta$ is associated with the decay constant f . The term ν^μ represents dissipative corrections (viscosity, resistance, etc.).

$$J_5^\mu = f^2 \partial^\mu \theta + \nu_5^\mu.$$

The term $f^2 \partial^\mu \theta$ is associated with the decay constant f . The term ν_5^μ represents dissipative corrections (viscosity, resistance, etc.).

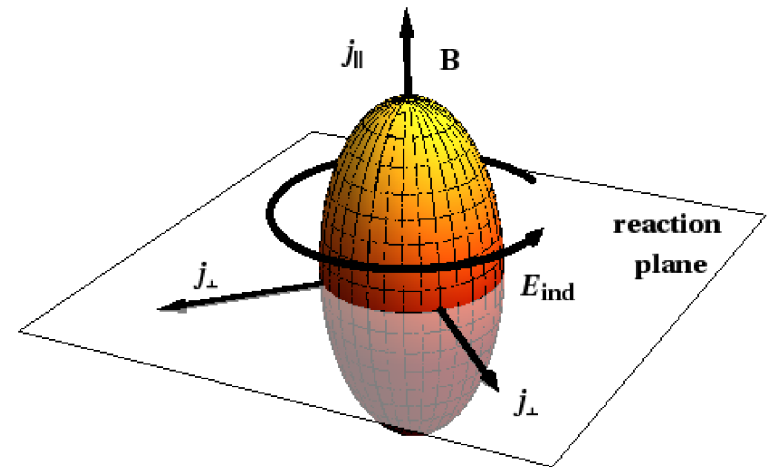
θ „decay constant“

Notice the additional current

Phenomenology

An additional electric current induced by the θ -field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta \cdot B)$$

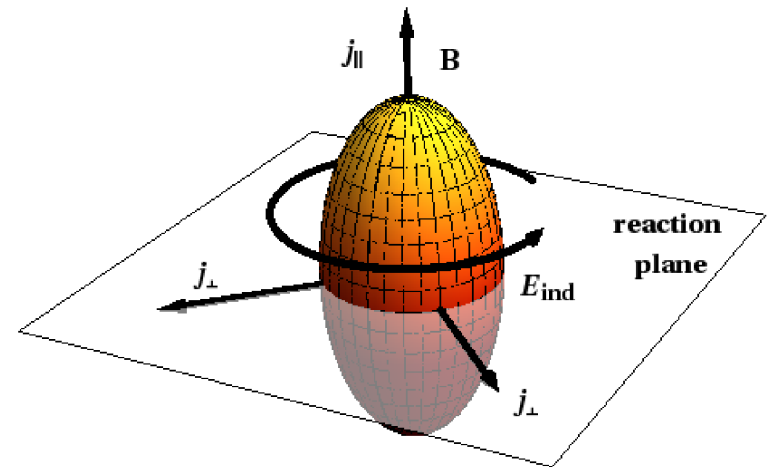


Phenomenology

An additional electric current induced by the θ -field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$


- **Chiral Magnetic Effect** (electric current along B-field)

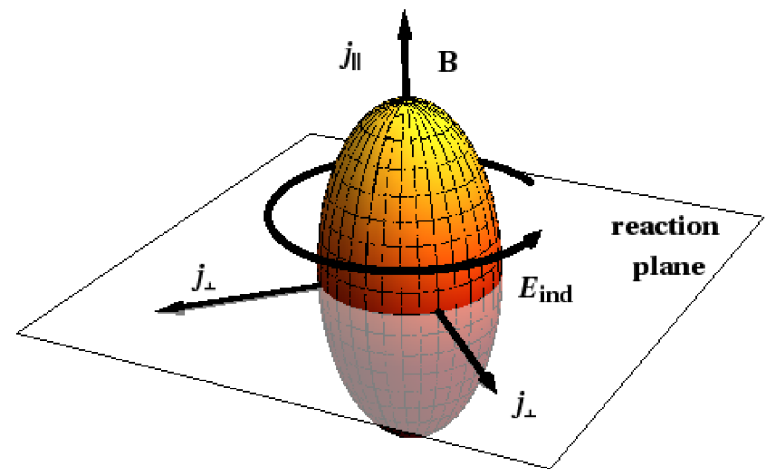


Phenomenology

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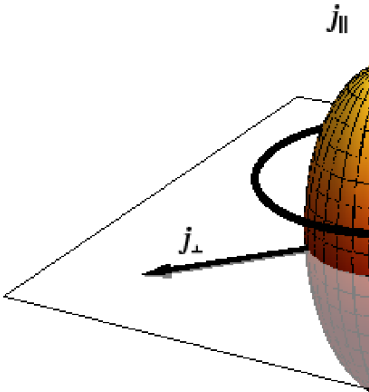
- **Chiral Magnetic Effect** (electric current along B-field)
 - **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- 

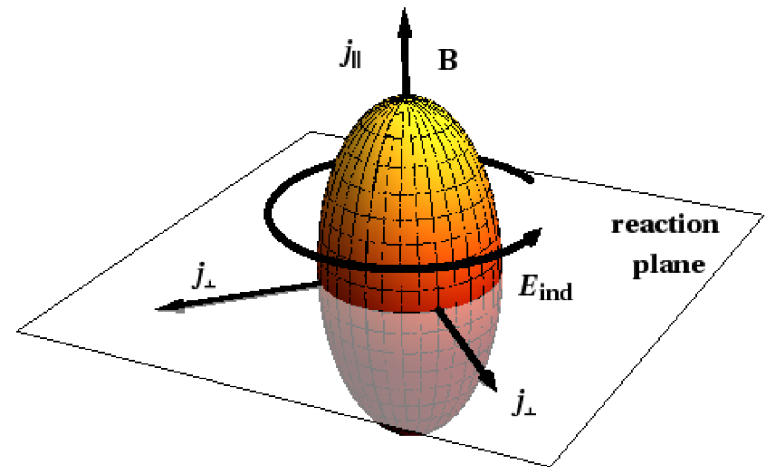


Phenomenology

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- **Chiral Magnetic Effect** (electric current along B-field)
 - **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
 - **Chiral Dipole Wave** (dipole moment induced by B-field)
- 
- The diagram shows a sphere with a grid pattern, representing a Fermi surface or a similar physical object. A vertical axis is labeled j_{\parallel} and a horizontal axis is labeled j_{\perp} , illustrating the Chiral Magnetic Effect.

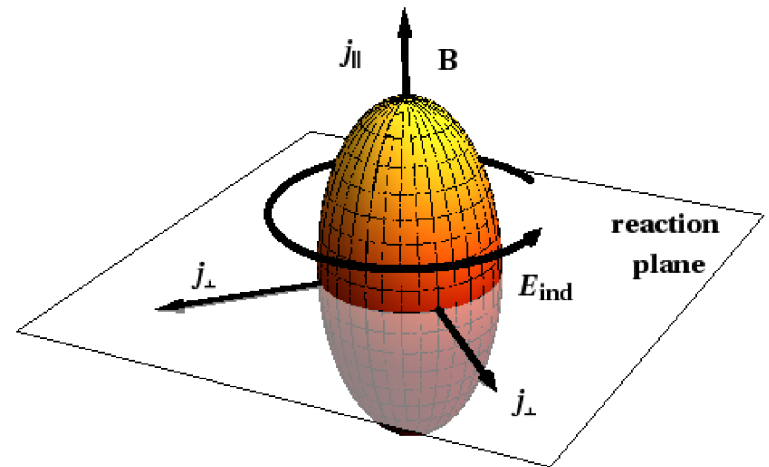


Phenomenology

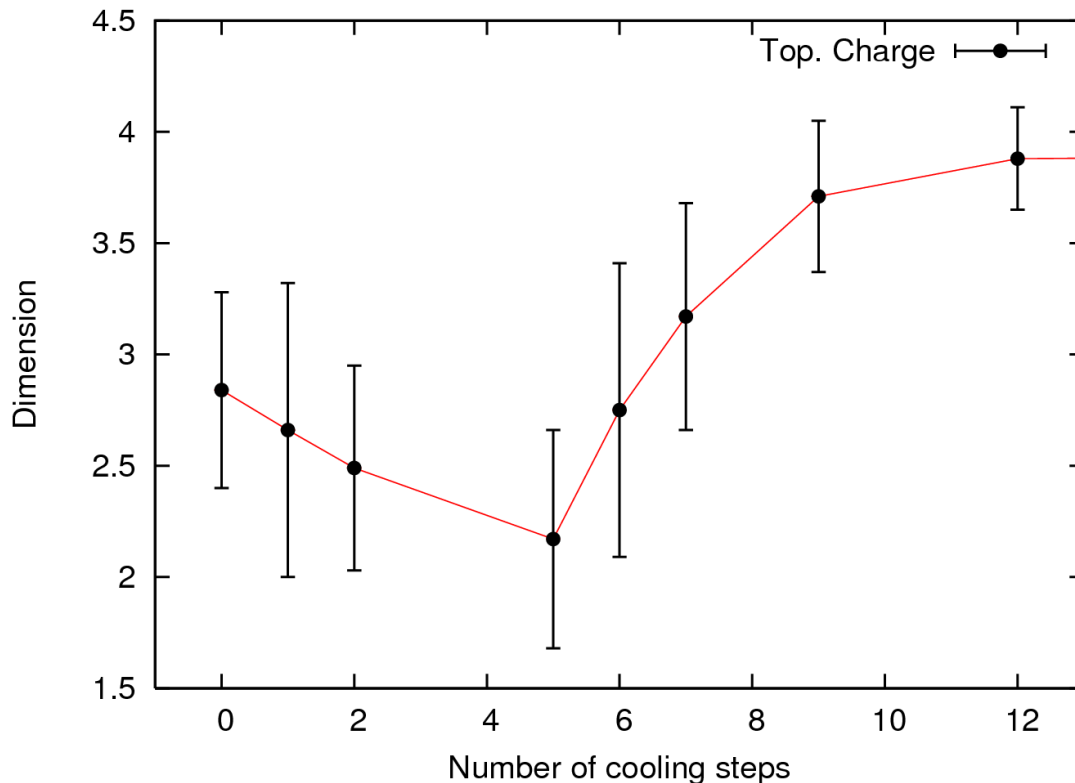
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- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)
- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field $\theta(x)$ itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



Fractal dimension



$$\text{IPR}(a) = \frac{\text{const}}{a^d}$$

Our result: **d = 2 ÷ 3**
and after cooling **d ~ 4**

d = 1: monopoles

d = 2: vortices

d = 3: domain walls

d = 4: instantons

$$\text{IPR} = \left\{ N \sum_x \rho_i^2(x) \mid \sum_x \rho_i(x) = 1 \right\}$$

Chromodynamic spaghetti

Still, the physical meaning of θ is not clear. It might be a field propagating along the percolating vortices (keep in mind $d=2..3$) without dissipation. We can test the color conductivity of QCD by solving the YM equations.

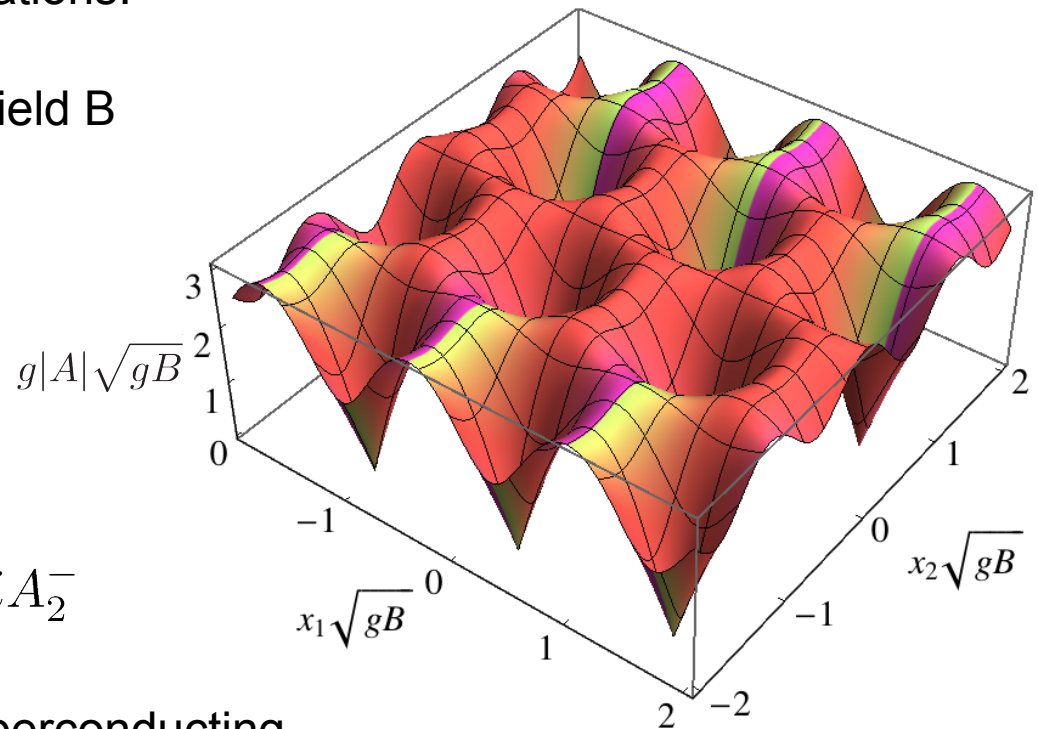
We switch on a constant chromomagnetic field B along the 3-rd spatial direction

$$A^3 = A_1^3 + iA_2^3 = \frac{B}{2} (ix_1 - x_2)$$

solve the YM equations for the transverse components

$$A_\mu^\pm = \frac{1}{\sqrt{2}} (A_\mu^1 \mp iA_\mu^2), \quad A = A_1^- + iA_2^-$$

and obtain the Abrikosov lattice of color-superconducting flux tubes. Fermionic zero modes will travel up and down along the Abrikosov vortices, depending on their chirality.



M. Chernodub, J. Van Doorselaere,
T.K., H. Verschelde,
Phys.Lett. B730 (2014) 63
and arXiv:1401.5974

Conclusions

- One should take into account low-dimensional defects, when dealing with rotation.
- The temperature corrections to the transport coefficients come from the statistics for the light chiral degrees of freedom.
- QCD in the range of temperatures $T_c < T < 2T_c$ can be described by a (non-conventional) chiral superfluid.
- The low-dimensional defects can also appear in QCD and "trap" light fermions.

**Thank you for the
attention!**