

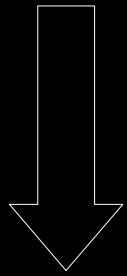
New approach to the local strong parity violation in the quark-gluon plasma



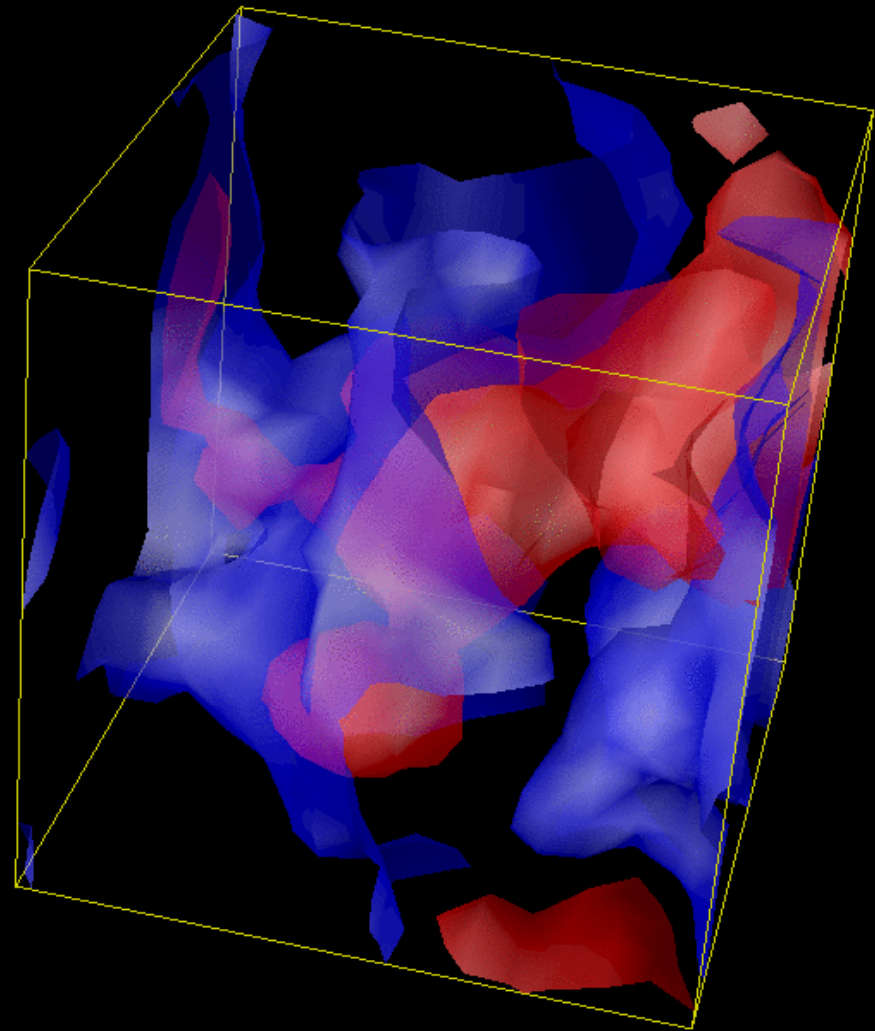
Tigran Kalaydzhyan

QCD vacuum

$$G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



$$\rho_R \neq \rho_L$$

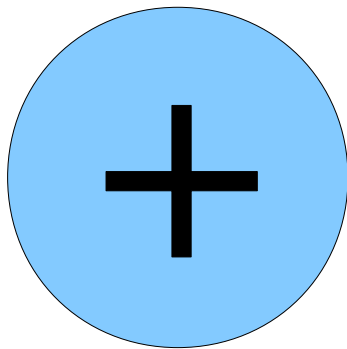


Positive topological
charge density

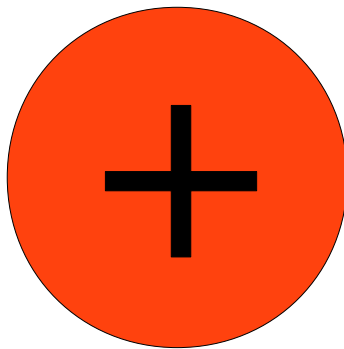
Negative topological
charge density

For the details of the simulation see P. Buividovich, T.K., M. Polikarpov PRD 86, 074511

(Naive) visible effects

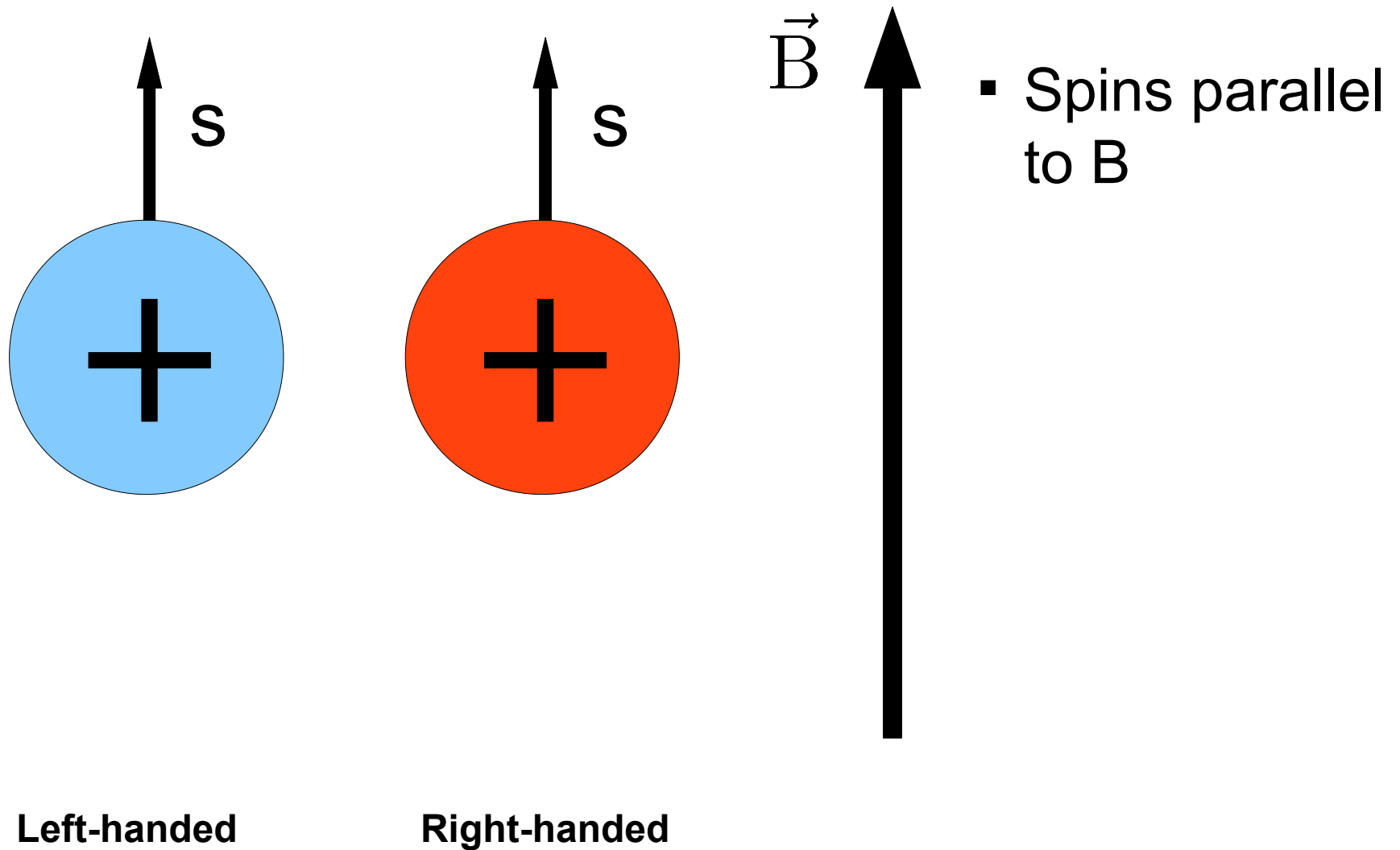


Left-handed

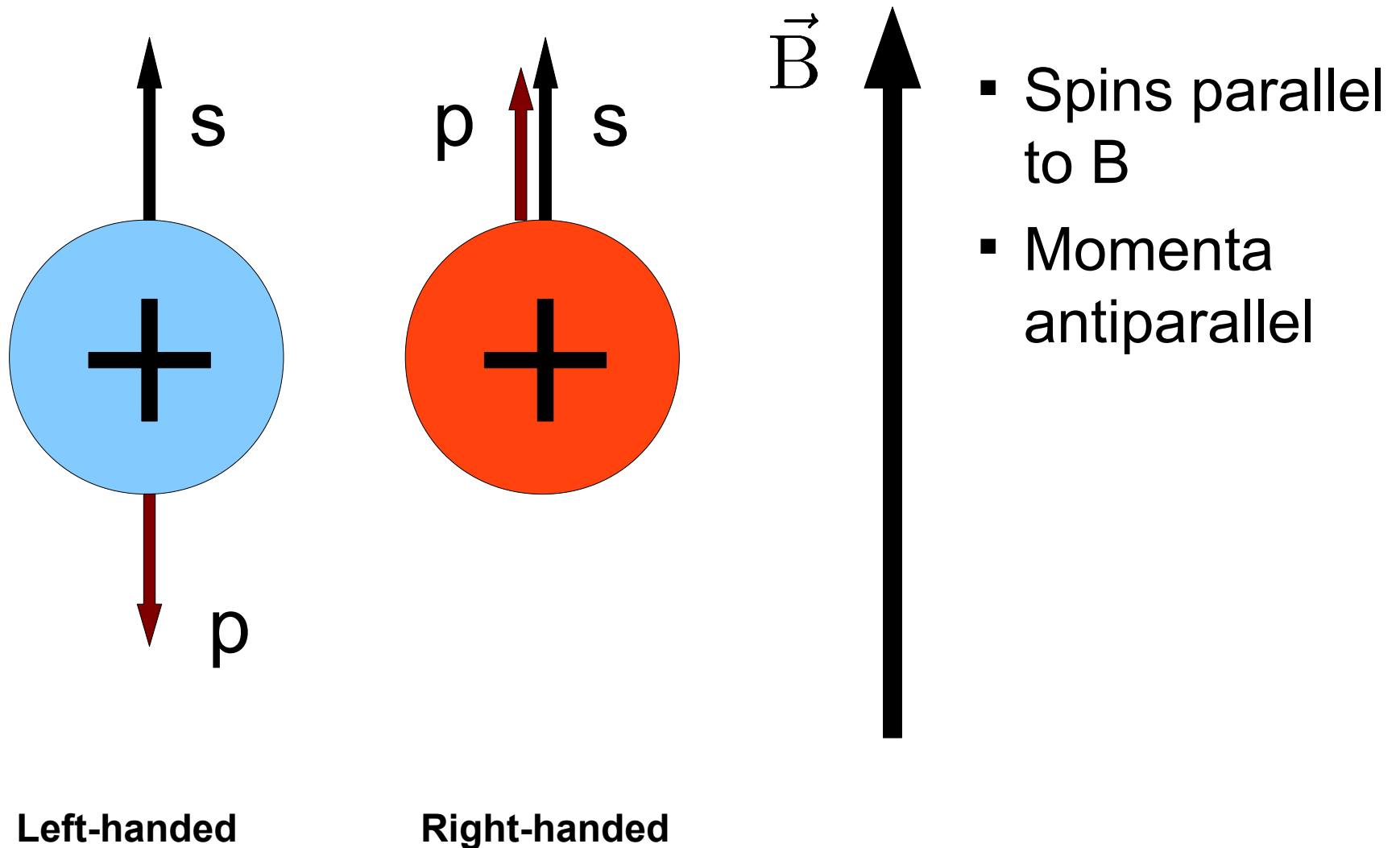


Right-handed

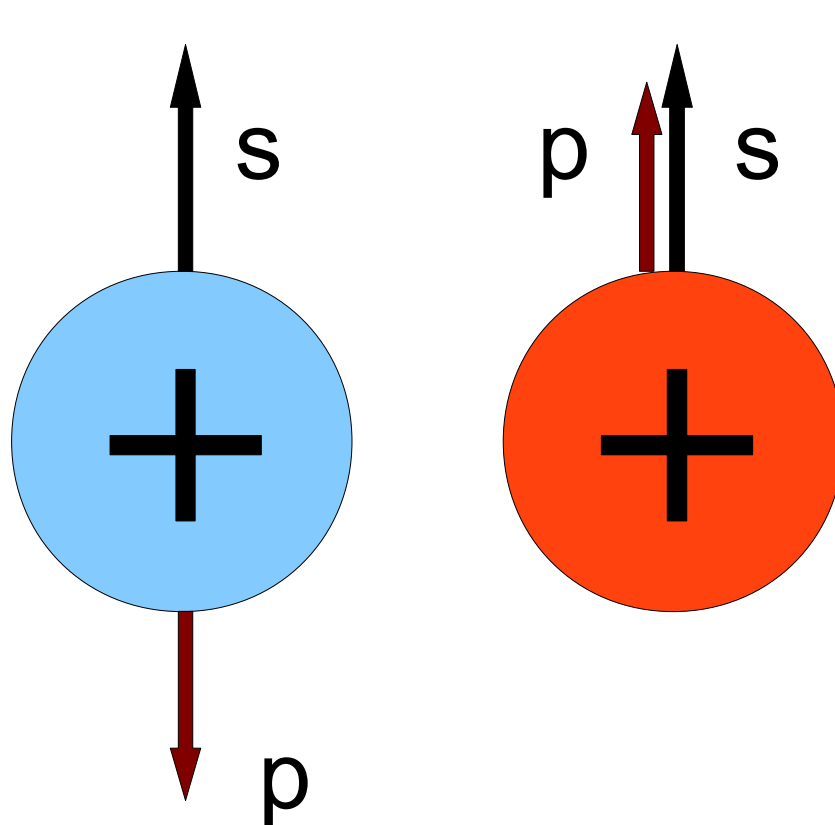
(Naive) visible effects



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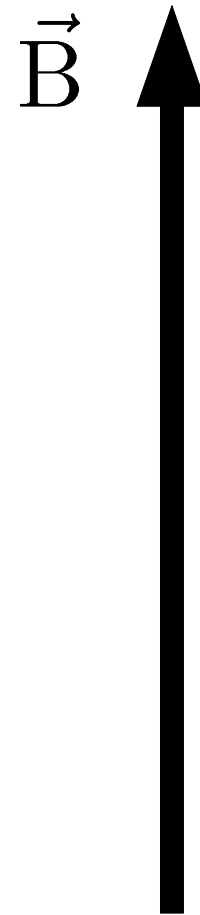


(Naive) visible effects



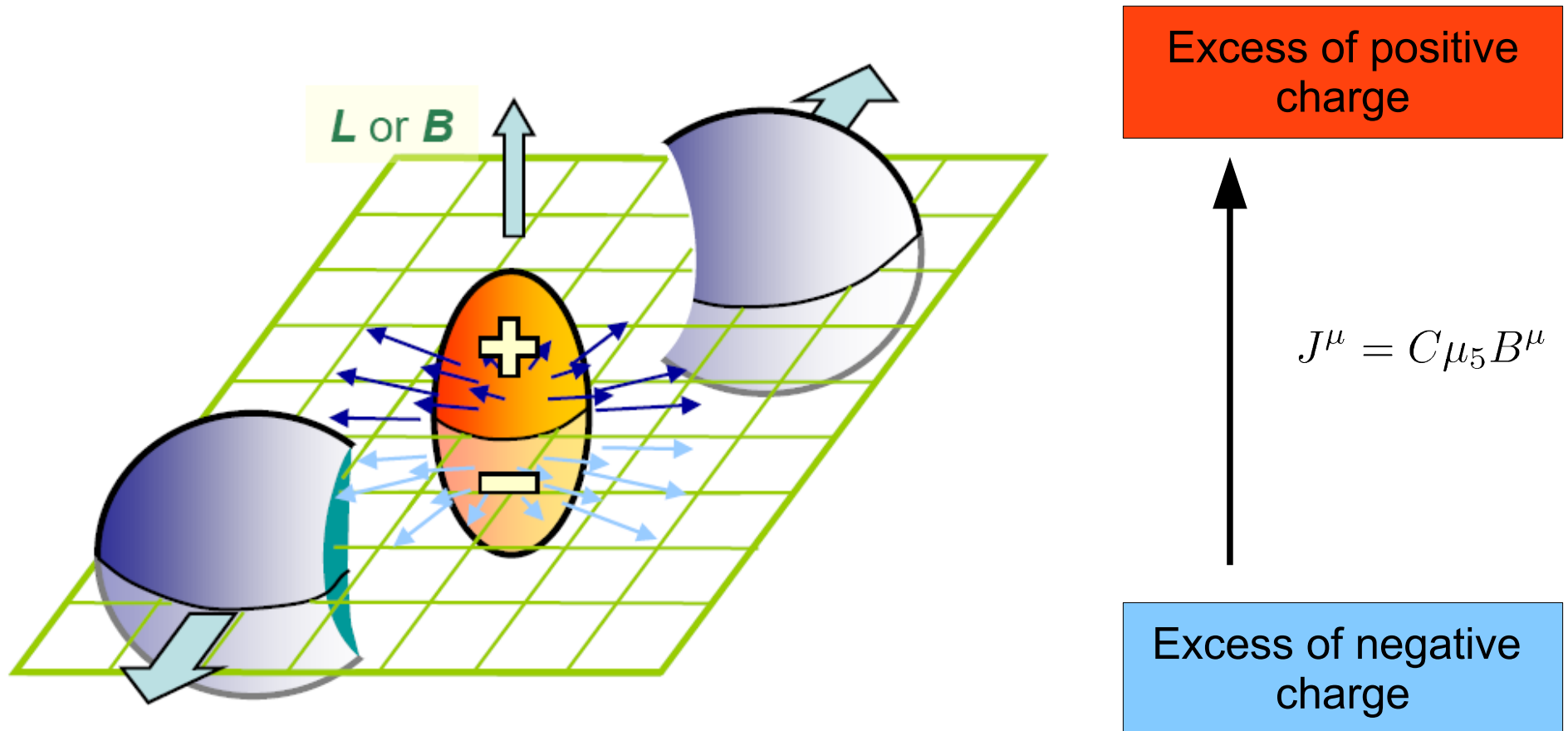
Left-handed

Right-handed



- Spins parallel to B
- Momenta antiparallel
- If $\rho_5 \equiv \rho_L - \rho_R \neq 0$ then we have a net electric current parallel to B

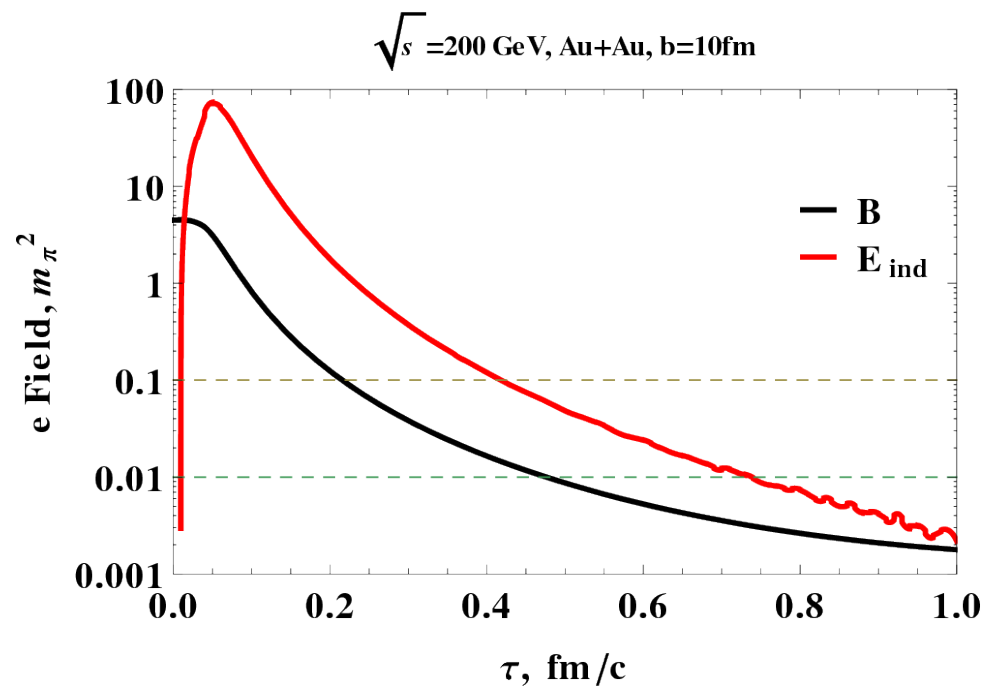
Chiral Magnetic Effect



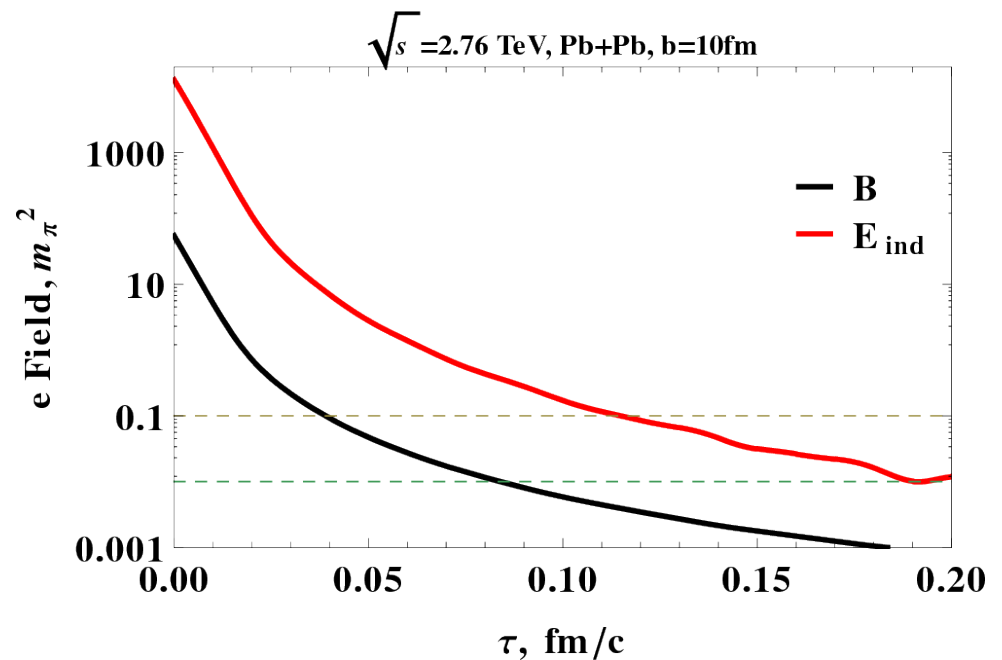
Fukushima, Kharzeev, McLerran, Warringa (2007)

For a local strong parity violation see e.g. 0909.1717 (STAR) and 1207.0900 (ALICE)

Electromagnetic fields



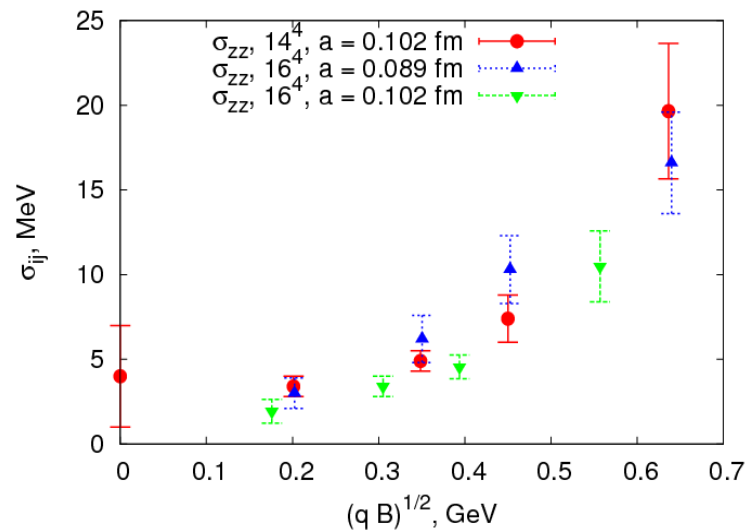
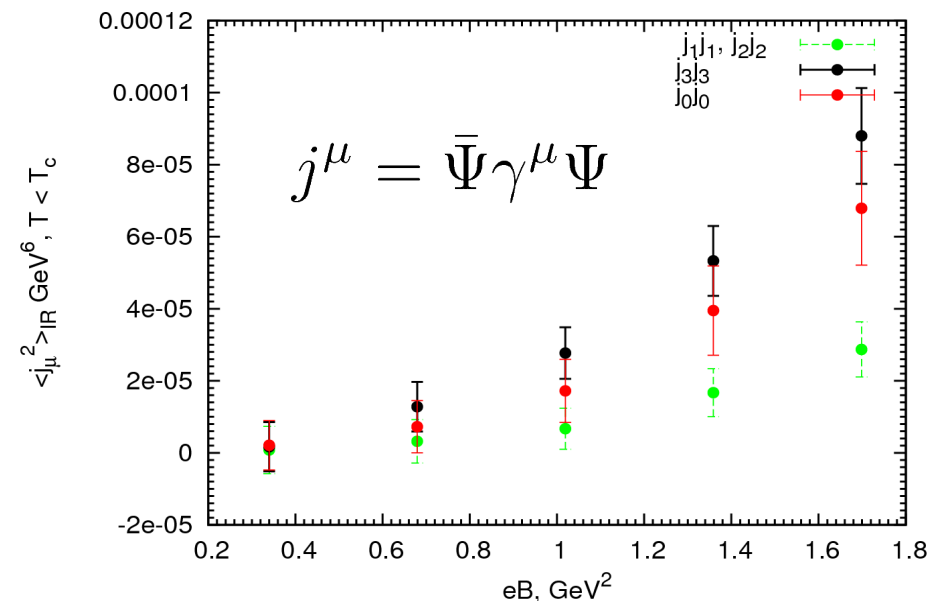
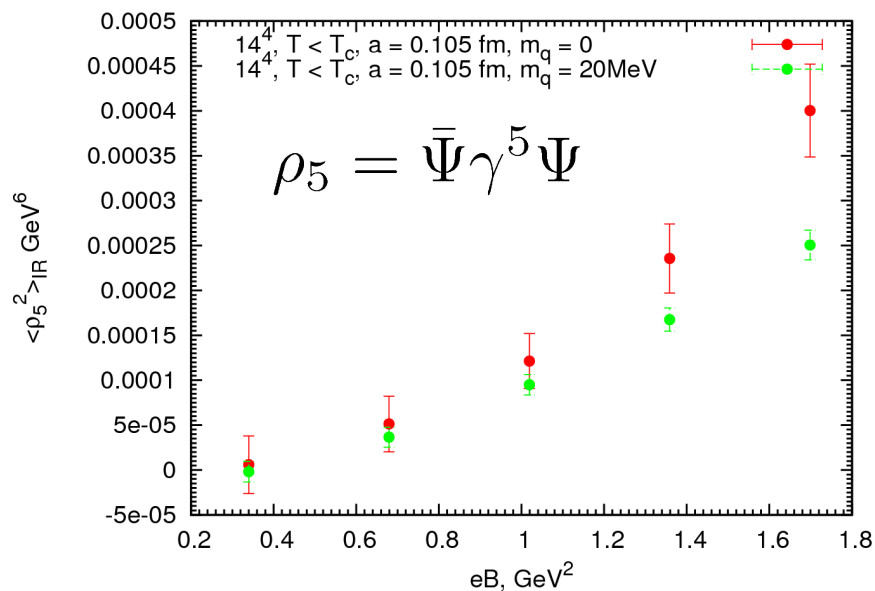
RHIC



LHC

Huge electromagnetic fields, never observed before!

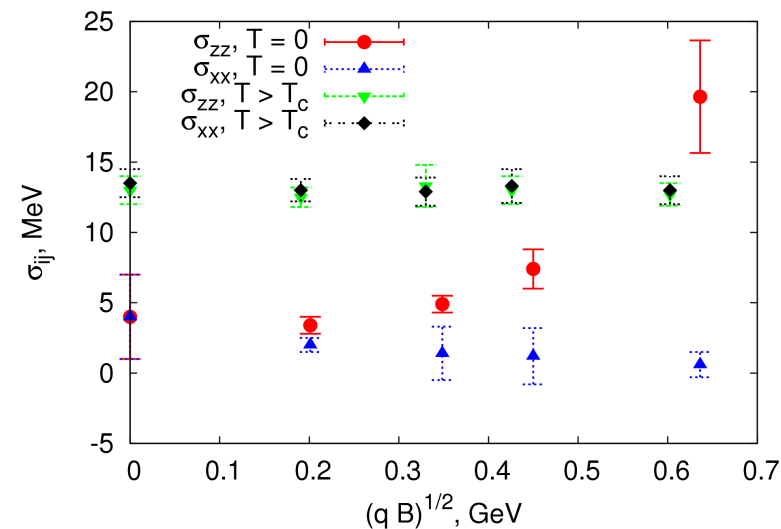
Some numbers (lattice)



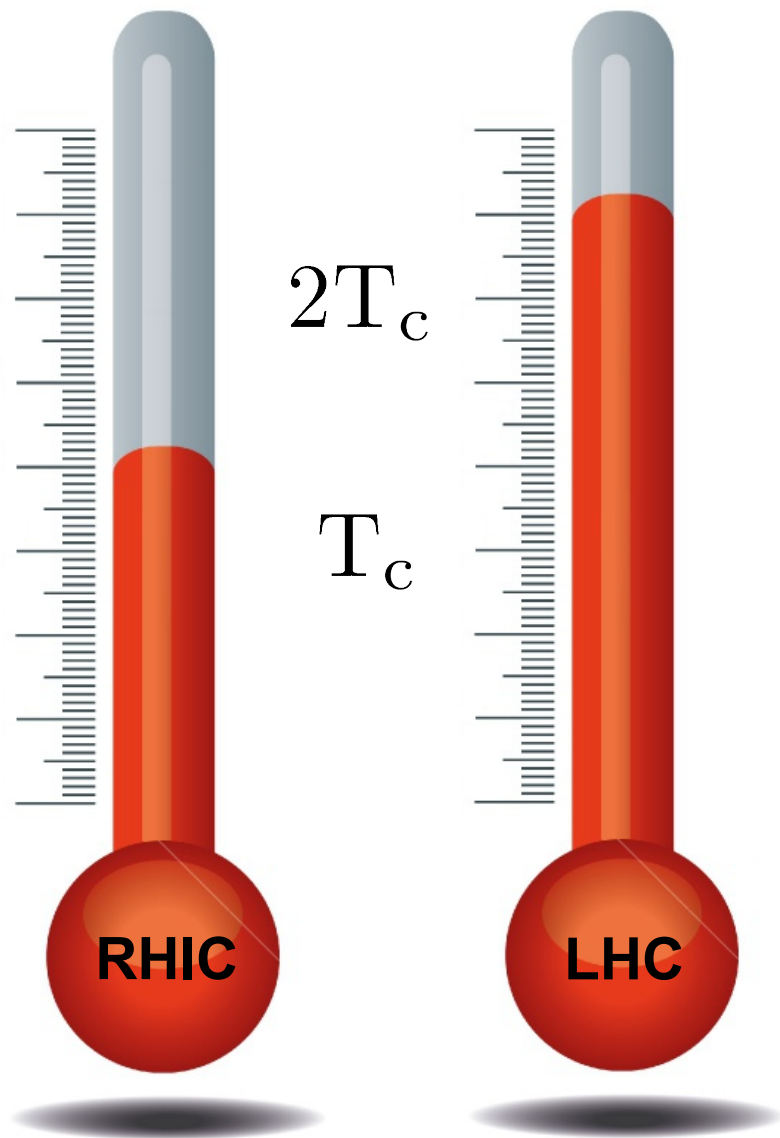
T.K., D. Kharzeev and



PRL 105 (2010) 132001
Phys.Atom.Nucl. 75, 488

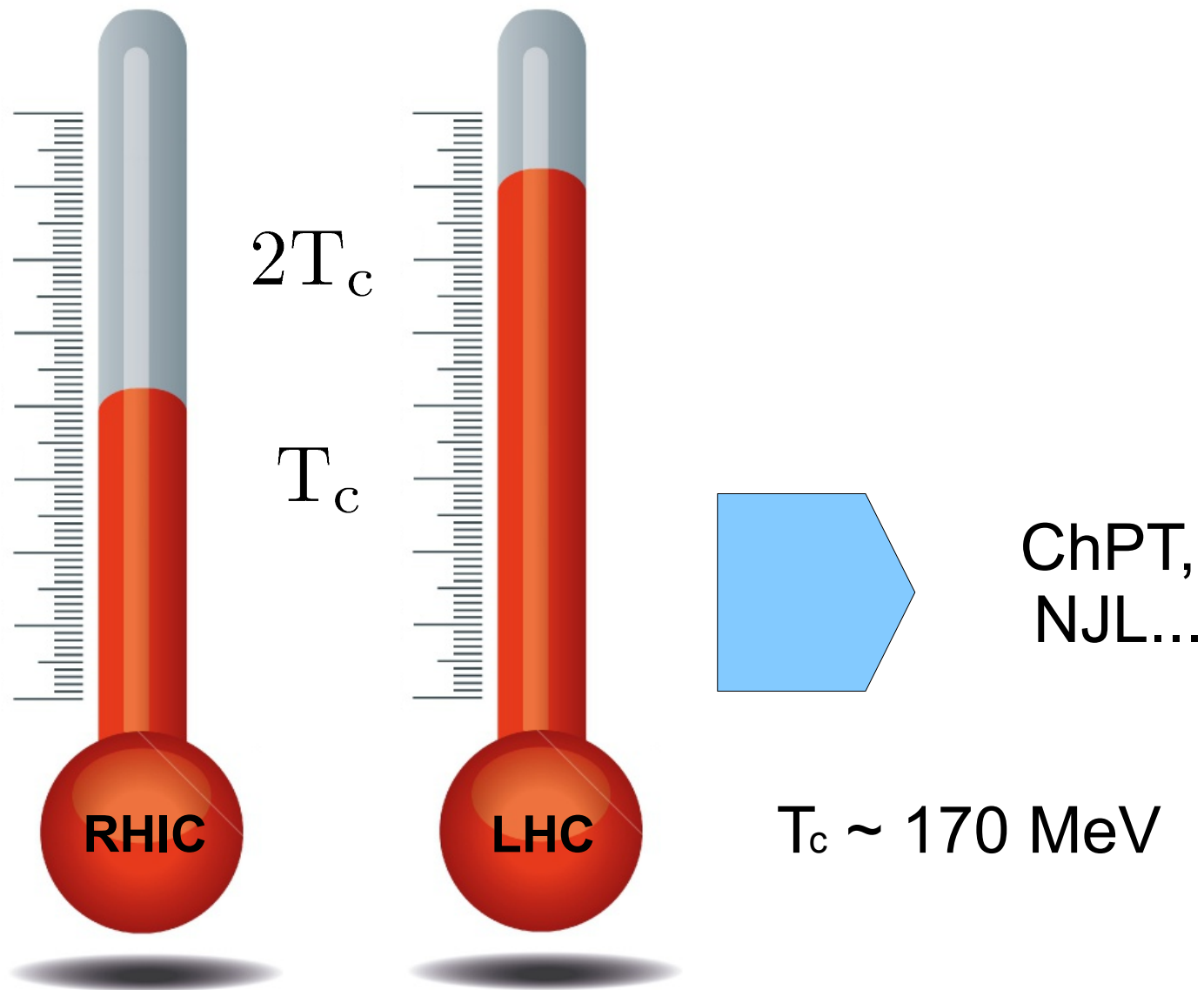


Temperatures

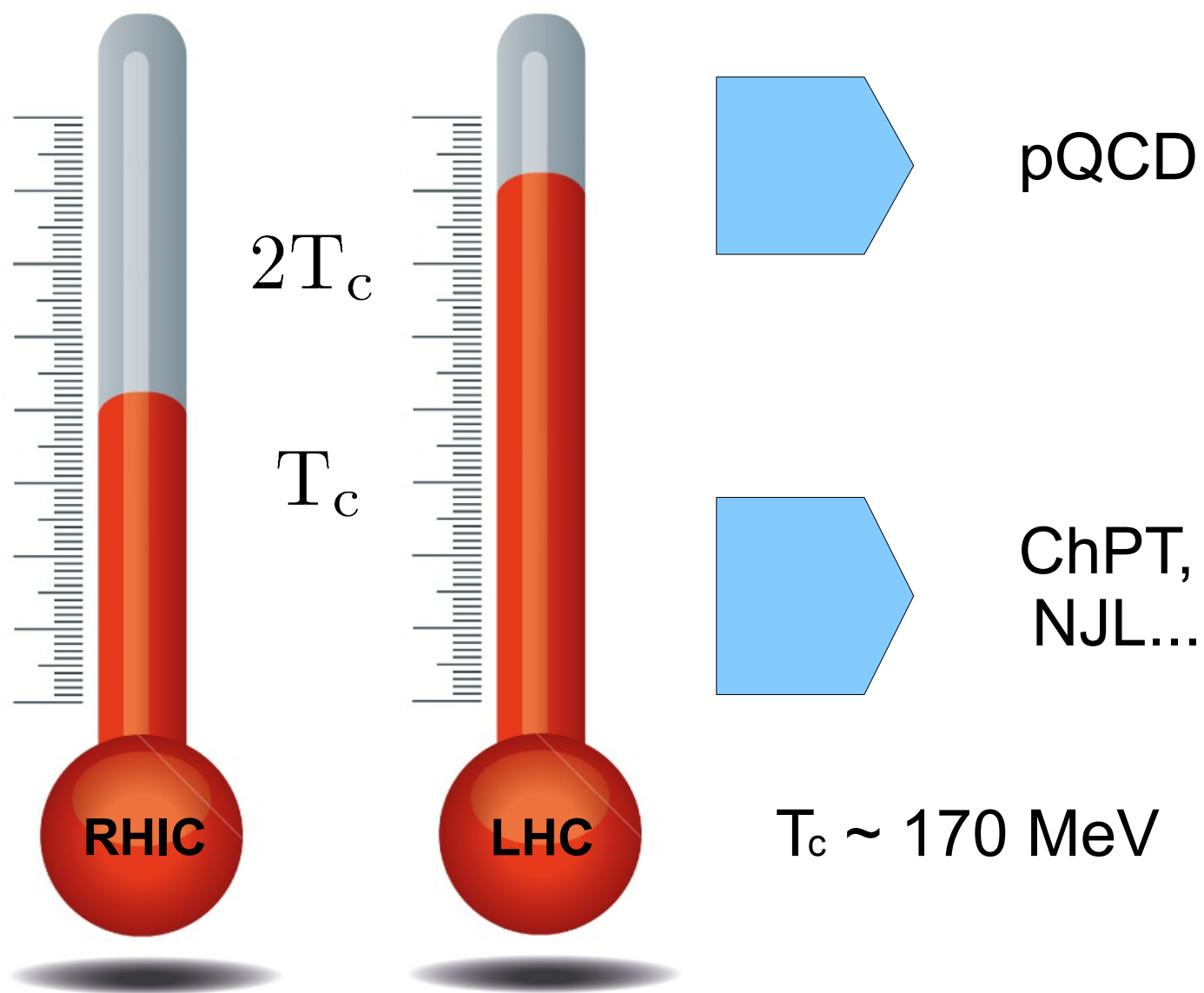


$T_c \sim 170 \text{ MeV}$

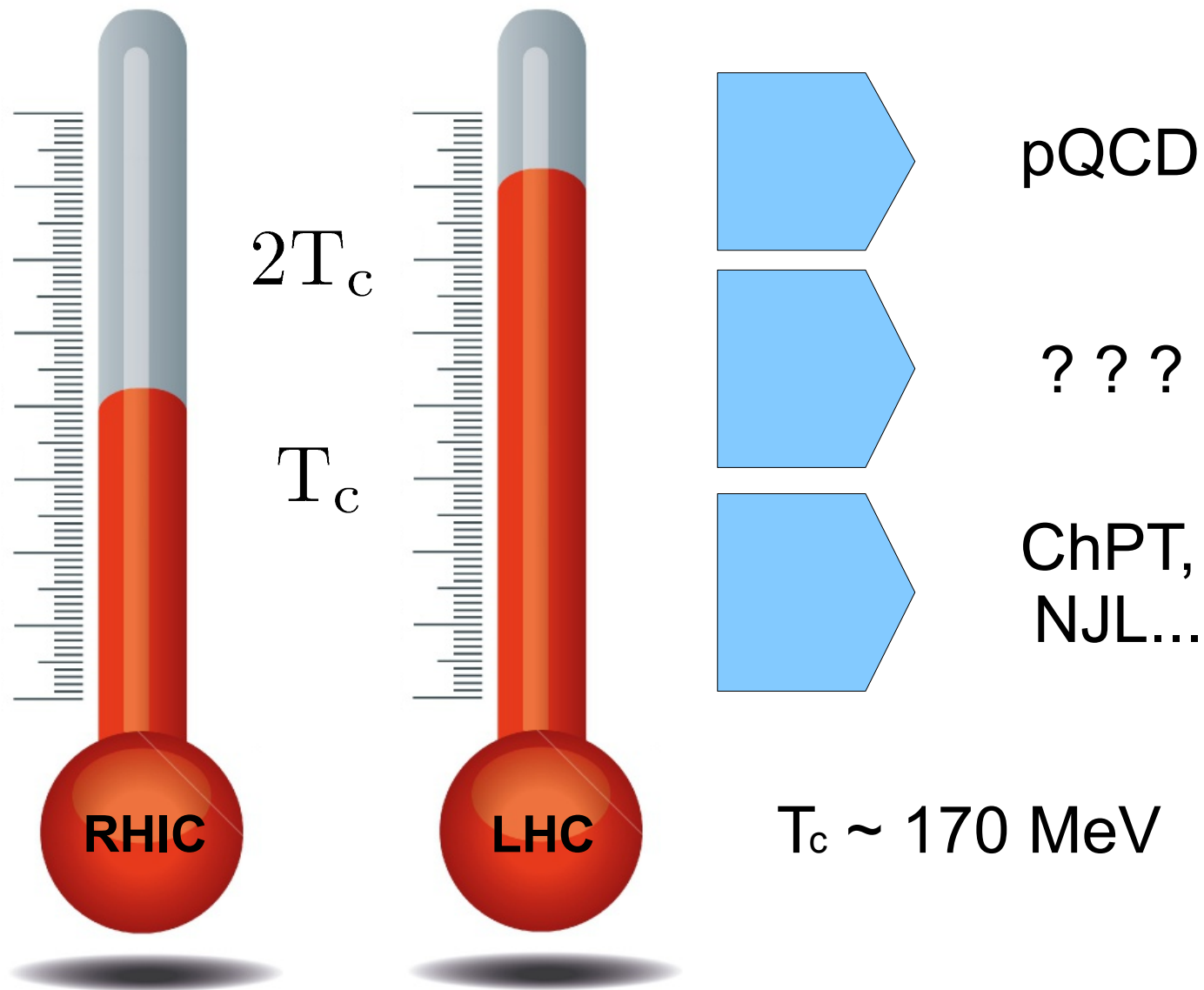
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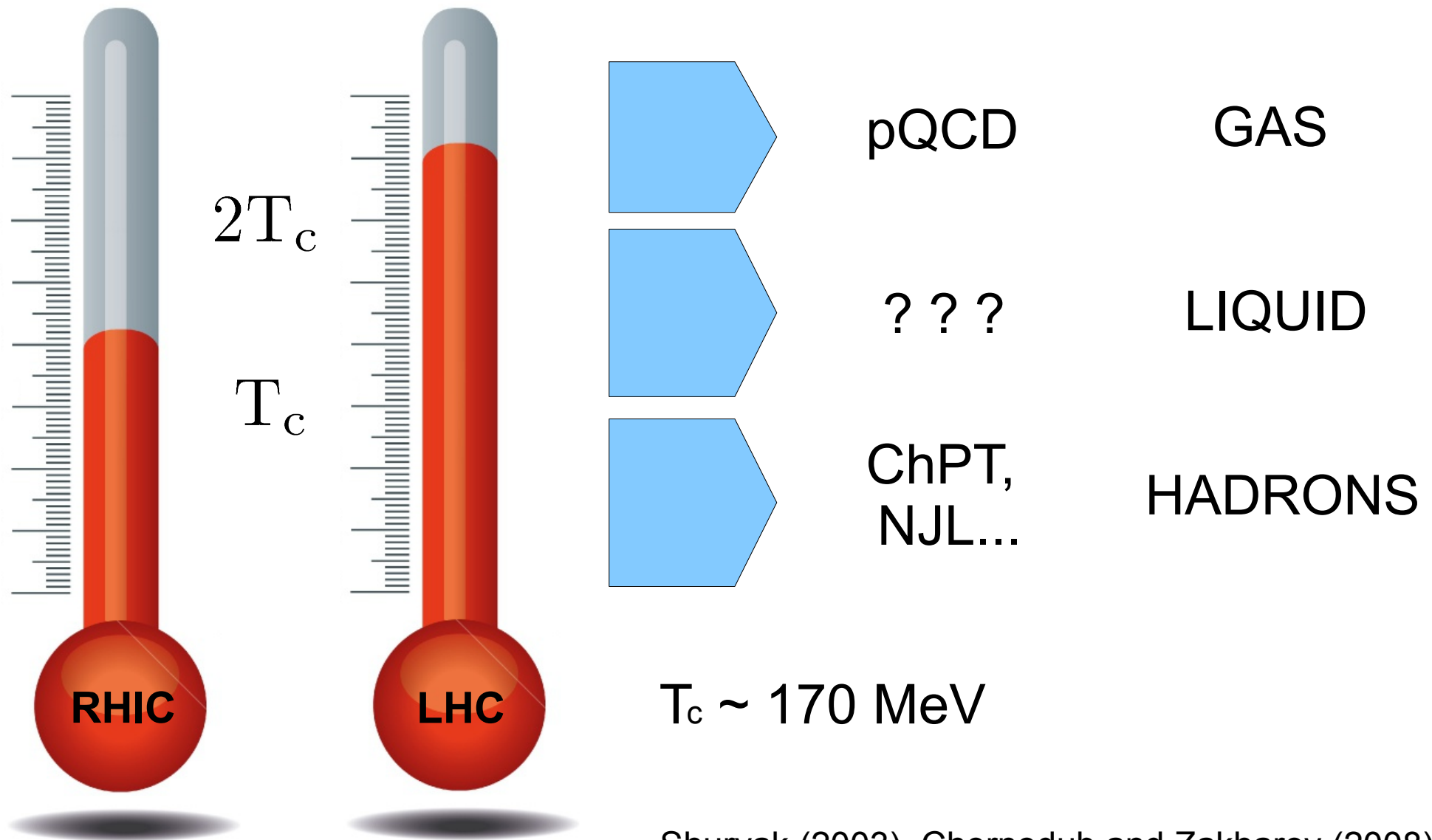
Temperatures



Temperatures



Temperatures



Shuryak (2003), Chernodub and Zakharov (2008)

Our task

- Build an effective model for QCD at $T_c < T < 2 T_c$ (did we lose anything?)

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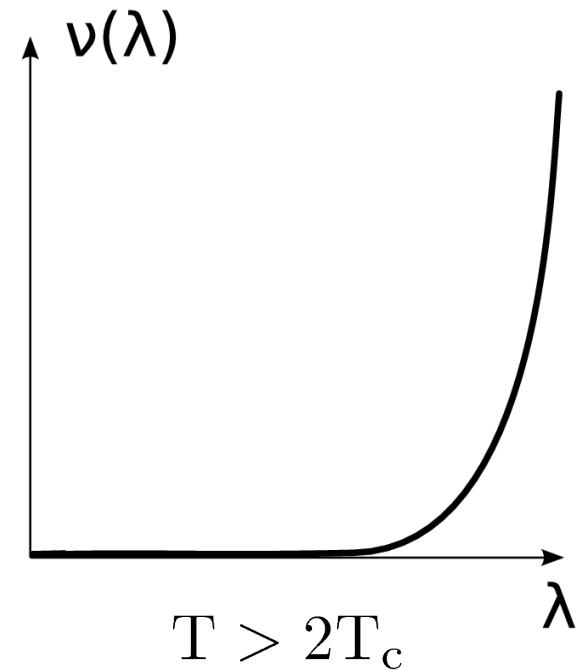
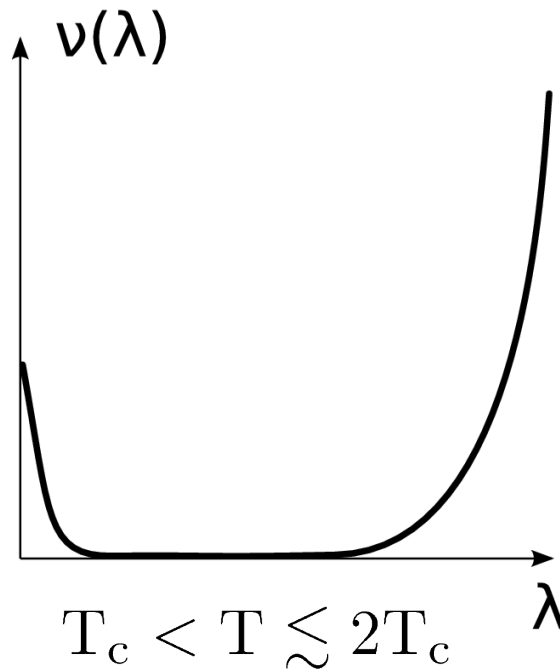
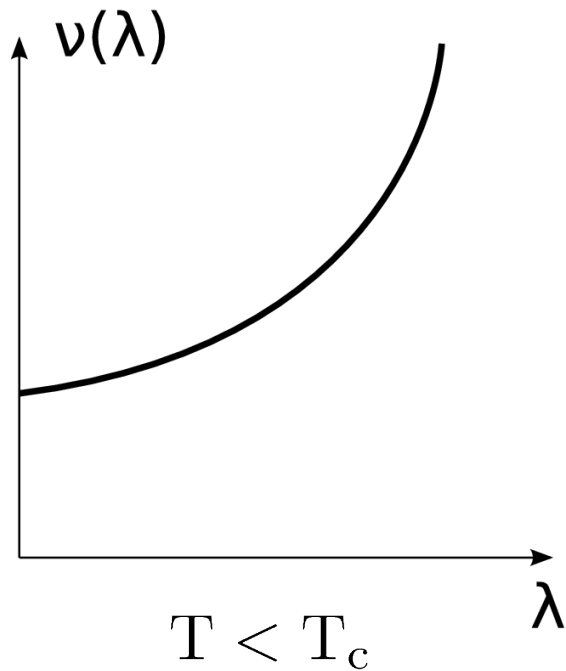
Our task

- Build an effective model for QCD at $T_c < T < 2 T_c$ (did we lose anything?)
- Find hydrodynamic equations corresponding to the effective Lagrangian
- Solve the hydro equations in gradient expansion, satisfying thermodynamic laws
- Extract phenomenological output for the heavy-ion collisions

Insight from the lattice

- Spectrum of the Dirac operator

$$\hat{D}\psi_\lambda = \lambda\psi_\lambda$$

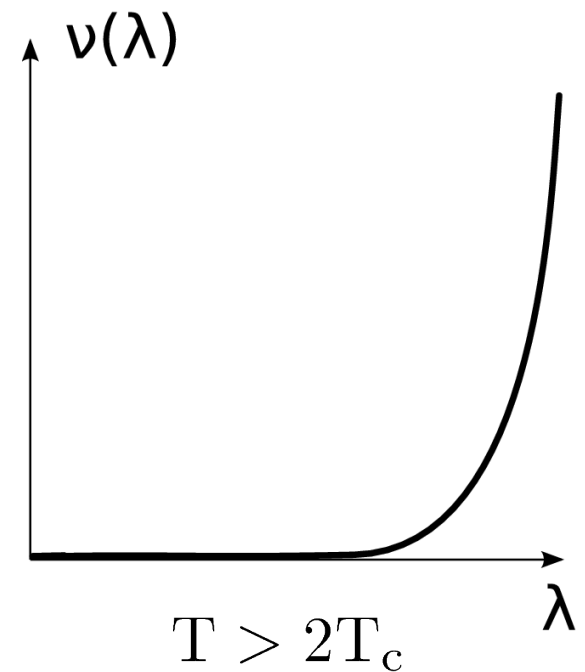
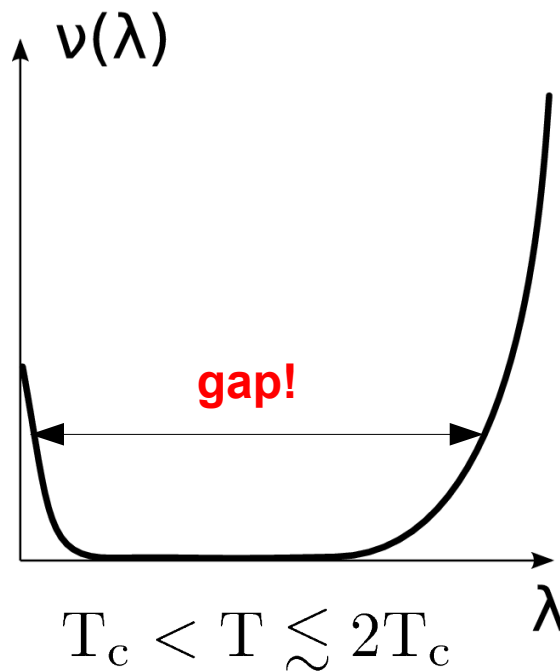
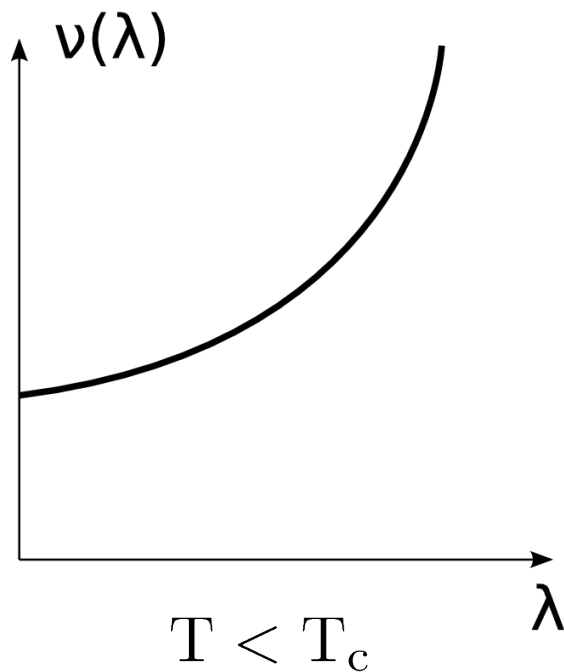


- Chiral properties are described by near-zero modes

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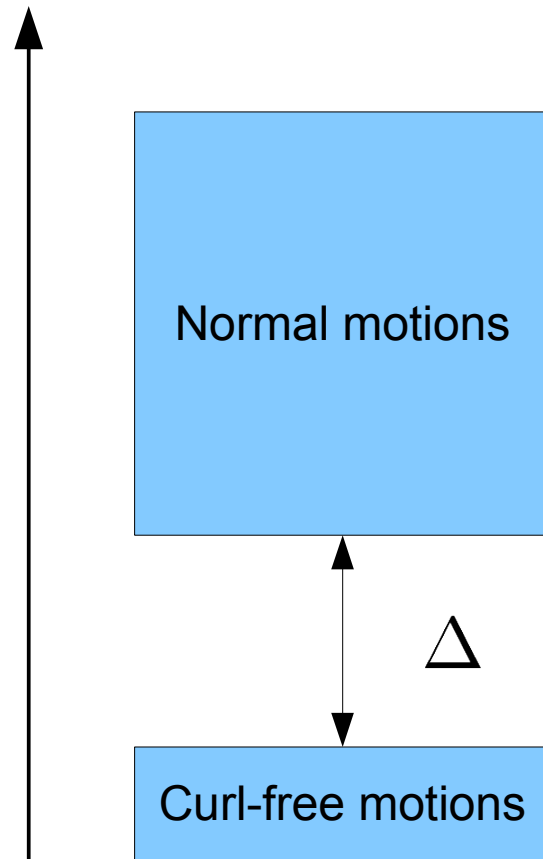
$$\hat{D}\psi_\lambda = \lambda\psi_\lambda$$



- Chiral properties are described by near-zero modes
- There are two separated parts of the spectrum at intermediate temperatures!

Why "superfluidity" ?

Energy



AUGUST 15, 1941

PHYSICAL REVIEW

Theory of the Superfluidity of Helium II

L. LANDAU

Institute of Physical Problems, Academy of Sciences USSR, Moscow, USSR

Therefore, between the lowest energy levels of vortex and potential motion there must be a certain energy interval Δ .

The supposition that the normal level of potential motions lies lower than the beginning of the spectrum of vortex motions leads to the phenomenon of superfluidity.

One of these motions is "normal" and the other is "superfluid."

We will not consider any spontaneously broken symmetry!

Bosonization

- Euclidean functional integral for $SU(N_c) \times U_{em}(1)$ is given by

$$\int D\bar{\psi} D\psi \exp \left\{ - \int_V d^4x \bar{\psi} (\not{D} - im) \psi + \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\},$$

where we define the Dirac operator as

$$\not{D} = -i(\not{\partial} + \not{A} + g\not{G} + \gamma_5 \not{A}_5).$$

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- and the chiral limit $m \rightarrow 0$

Bosonization

- The **total effective Euclidean Lagrangian** reads as

$$\begin{aligned}\mathcal{L}_E^{(4)} = & \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu \\ & + \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{g^2}{16\pi^2} \theta G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + \frac{N_c}{8\pi^2} \theta F^{\mu\nu} \tilde{F}_{\mu\nu} \\ & + \frac{N_c}{24\pi^2} \theta \square^2 \theta - \frac{N_c}{12\pi^2} (\partial^\mu \theta \partial_\mu \theta)^2\end{aligned}$$

So we get an axion-like field with decay constant $f = \frac{2\Lambda}{\pi} \sqrt{N_c}$

and a negligible mass $m_\theta^2 = \lim_{V \rightarrow \infty} \frac{\langle Q^2 \rangle}{f^2 V} \equiv \chi(T)/f^2$.

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Dynamical axion-like internal degree of freedom in QCD!

Interpretation of the scale Λ

- From the quartic Lagrangian at $N_c = N_f = 1$ we get

$$\rho_5 = - \lim_{t_E \rightarrow 0} \frac{\delta \mathcal{L}_E^{(4)}}{\delta \mu_5} = \frac{1}{2} \left(\frac{\Lambda}{\pi} \right)^2 \mu_5 + \frac{1}{3\pi^2} \mu_5^3$$

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- Free quarks and a strong B-field: $\Lambda = 2\sqrt{|eB|}$
- Dynamical fermions (1105.0385): $\Lambda \simeq 3 \text{ GeV} \gg \Lambda_{QCD}$

A „hidden“ non-perturbative scale!

One more remark

„Axionic“ part of the Lagrangian

$$\mathcal{L}_\theta = \frac{\Lambda^2 N_c}{4\pi^2} \partial^\mu \theta \partial_\mu \theta + \frac{N_c}{24\pi^2} \theta \square^2 \theta - \frac{N_c}{12\pi^2} (\partial^\mu \theta \partial_\mu \theta)^2 + \dots$$

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If magnetic field dominates over other scales, then we can make the following redefinition:

$$\theta \rightarrow \frac{\pi}{\sqrt{2N_c e B}} \theta$$

$$\mathcal{L}_\theta \rightarrow \frac{1}{2} \partial^\mu \theta \partial_\mu \theta + \frac{1}{48 \textcolor{red}{eB}} \theta \square^2 \theta - \frac{\pi^2}{48 N_c (\textcolor{red}{eB})^2} (\partial^\mu \theta \partial_\mu \theta)^2 + \dots$$

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In the limit $B \rightarrow \infty$ bosonization becomes exact, which is an evidence of the $(3+1) \rightarrow (1+1)$ reduction!

Hydrodynamic equations

Considering EOM for the Minkowski effective Lagrangian and only the color-singlet states, we obtain:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda ,$$

$$\partial_\mu J^\mu = 0 ,$$

$$\partial_\mu J_5^\mu = C E^\mu B_\mu ,$$

$$u^\mu \partial_\mu \theta + \mu_5 = 0 ,$$

Hydrodynamic equations

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Energy-momentum tensor $\rightarrow \partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda ,$

$\partial_\mu J^\mu = 0 ,$ Total electric current

Axial current $\rightarrow \partial_\mu J_5^\mu = C E^\mu B_\mu ,$ Electromagnetic fields

Chiral anomaly coefficient

4-velocity of the normal component $\rightarrow u^\mu \partial_\mu \theta + \mu_5 = 0 ,$ Josephson equation, defining The axial chemical potential through the bosonized low-lying modes.

Similar to the superfluid dynamics!

Hydrodynamic equations

- Solving the hydrodynamic equations in the gradient expansion, we obtain the constitutive relations:

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu},$$

$$J^\mu = \rho u^\mu + C \tilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu,$$

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Hydrodynamic equations

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The diagram illustrates the constitutive relations for hydrodynamic quantities. It features three equations with arrows pointing from descriptive labels to specific terms within them:

- $T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + f^2 \partial^\mu \theta \partial^\nu \theta + \tau^{\mu\nu},$
 - Energy density ϵ is associated with the $(\epsilon + P)$ term.
 - Pressure P is associated with the P term in $P g^{\mu\nu}$.
- $J^\mu = \rho u^\mu + C \tilde{F}^{\mu\kappa} \partial_\kappa \theta + \nu^\mu,$
 - Charge density ρ is associated with the ρu^μ term.
- $J_5^\mu = f^2 \partial^\mu \theta + \nu_5^\mu.$
 - θ „decay constant“ is associated with the $\partial^\mu \theta$ term.

Arrows from the right side of the equations point to a common label: Dissipative corrections (viscosity, resistance, etc.), which refers to the $\tau^{\mu\nu}$, ν^μ , and ν_5^μ terms.

Notice the additional current

Change in entropy and higher order gradient corrections

Higher order correction obey the Landau conditions

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Using both hydrodynamic equations and constitutive relations one can derive

$$\partial_\mu \left(s u^\mu - \frac{\mu}{T} \nu^\mu - \frac{\mu_5}{T} \nu_5^\mu \right) = -\frac{1}{T} (\partial_\mu u_\nu) \tau^{\mu\nu} - \nu^\mu \left(\partial_\mu \frac{\mu}{T} - \frac{1}{T} E_\mu \right) - \nu_5^\mu \partial_\mu \frac{\mu_5}{T}$$

- Entropy production is always non-negative

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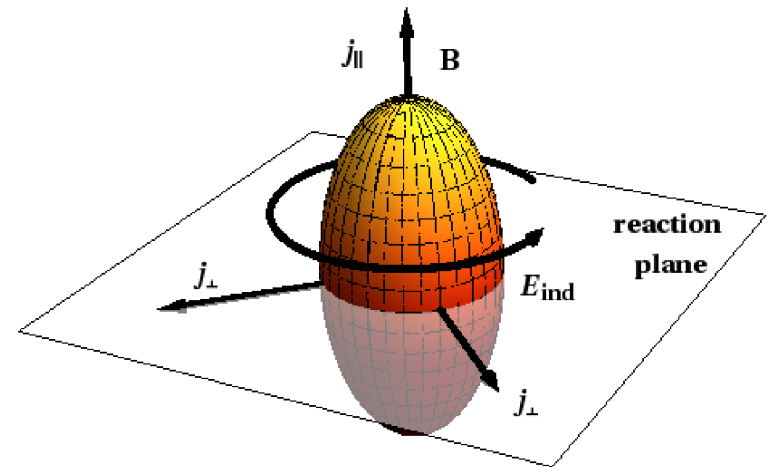
$$\partial_\mu (su^\mu - \cancel{\frac{\mu}{T}} - \cancel{\frac{\mu_5}{T}}) = -\frac{1}{T}(\partial_\mu \cancel{\nu^\mu})\tau^{\mu\nu} - \cancel{\nu^\mu}(\partial_\mu \cancel{\frac{\mu}{T}} - \frac{1}{T}E_\mu) - \cancel{\nu_5^\mu} \cancel{\partial_\mu \frac{\mu_5}{T}}$$

- Entropy production is always non-negative
- No additional anomalous first-order corrections to the currents
- Only the “normal” component contributes to the entropy current, while the “superfluid” component has zero entropy

Phenomenology

An additional electric current induced by the θ -field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta \cdot B)$$

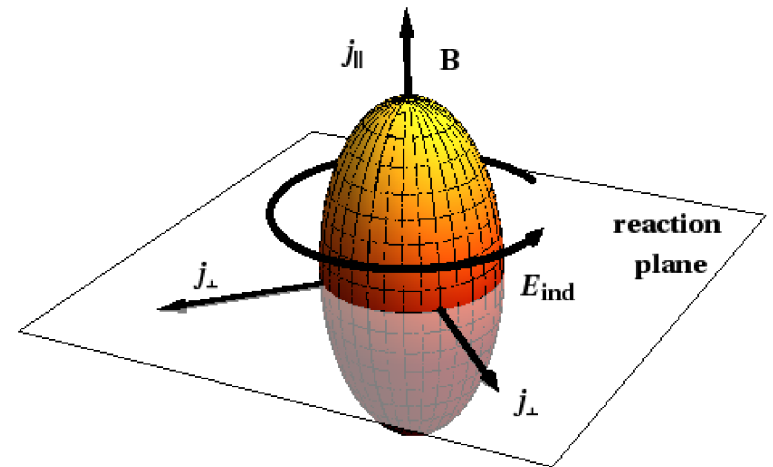


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- **Chiral Magnetic Effect** (electric current along B-field)

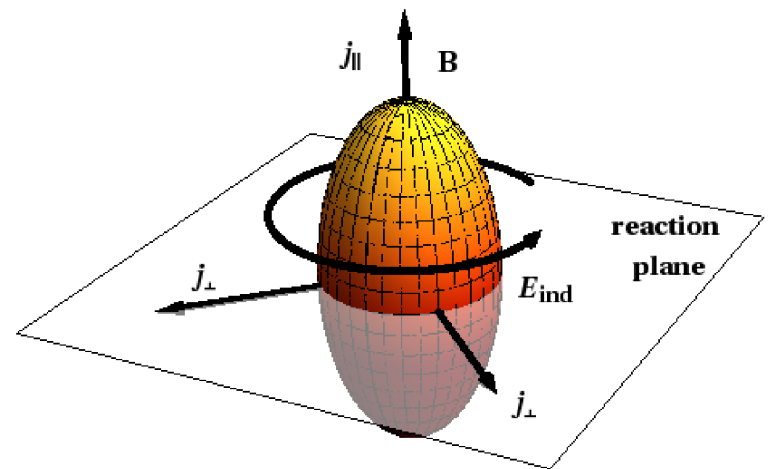


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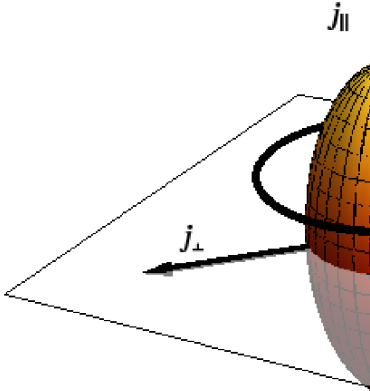
- **Chiral Magnetic Effect** (electric current along B-field)
- **Chiral Electric Effect** (electric current transverse to E-field and to both normal and superfluid velocities)

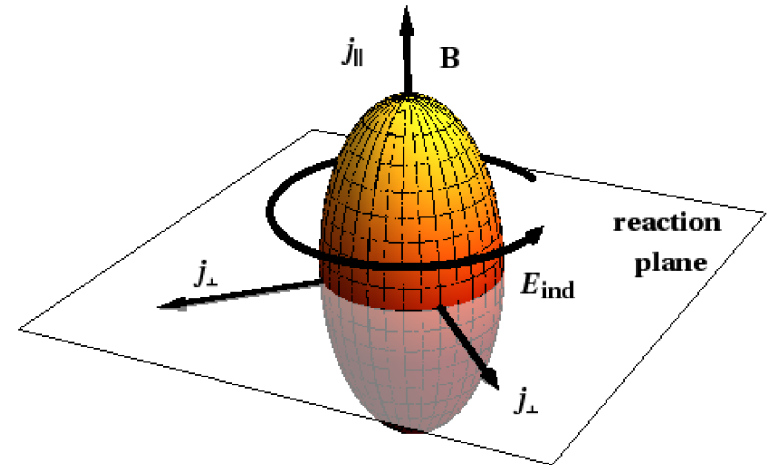


Phenomenology

An additional electric current induced by the θ -field:

$$j_\lambda = -C\mu_5 B_\lambda + C\epsilon_{\lambda\alpha\kappa\beta}u^\alpha\partial^\kappa\theta E^\beta - Cu_\lambda(\partial\theta\cdot B)$$

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 - **Chiral Dipole Wave** (dipole moment induced by B-field)
- 
- The diagram shows a sphere with a grid pattern, representing a Fermi surface or a similar physical structure. A vertical axis is labeled $j_{\parallel}$, and a horizontal axis is labeled $j_{\perp}$. A curved arrow on the sphere's surface indicates a direction of flow or rotation.

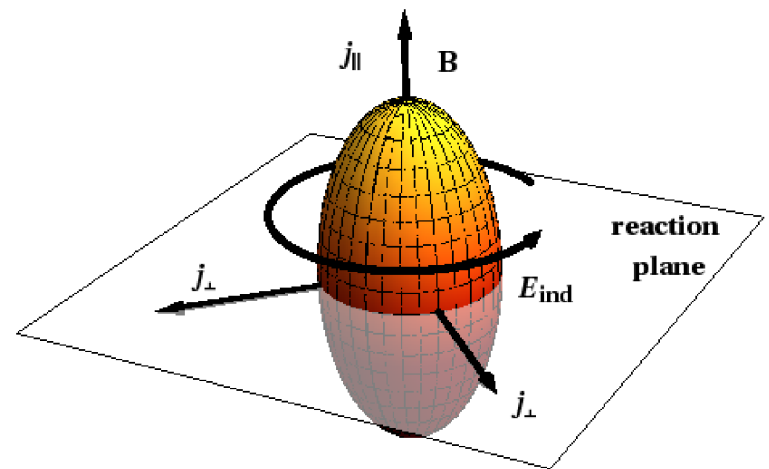


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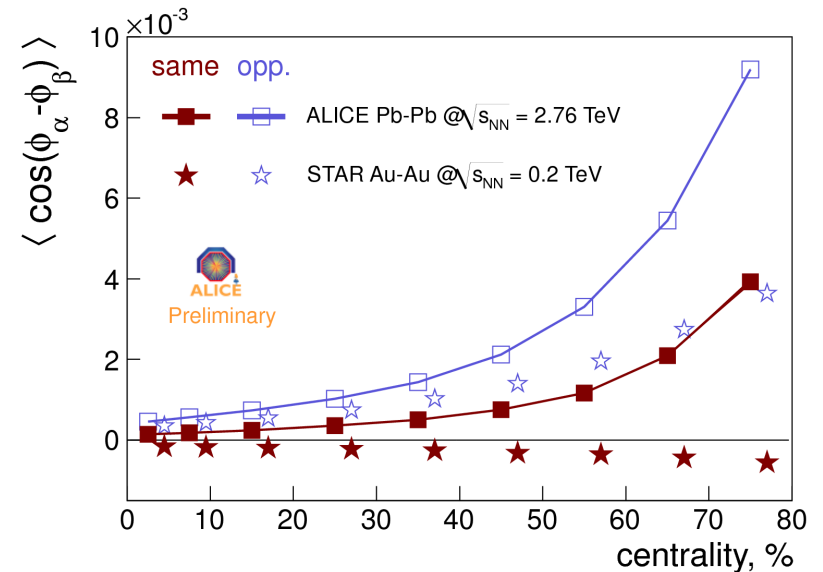
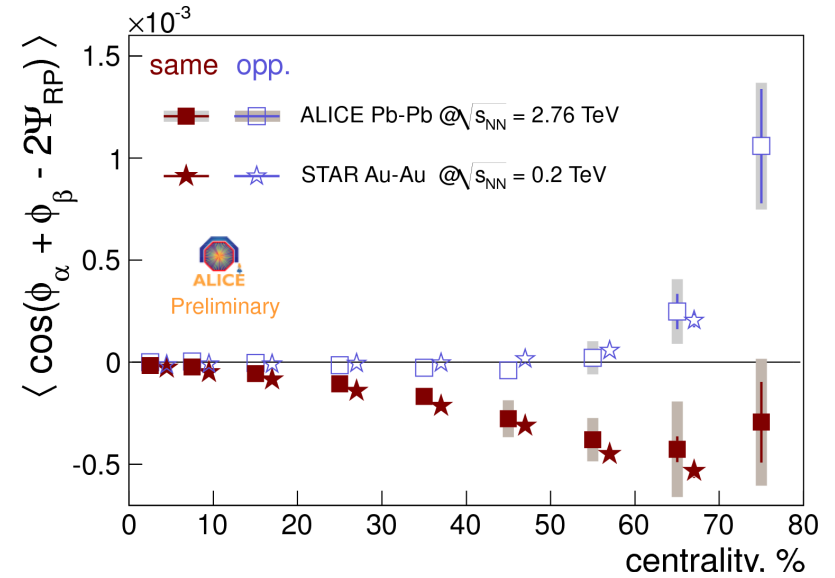
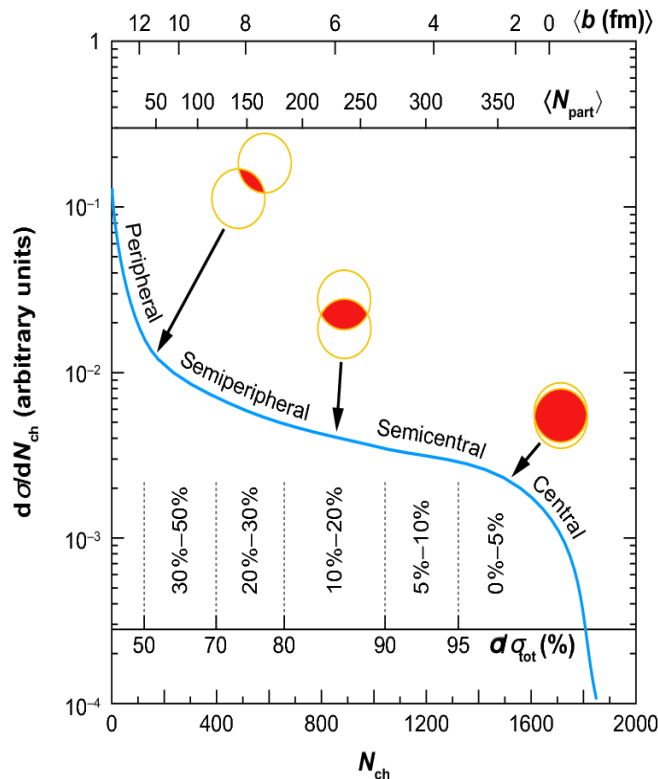
- **Chiral Magnetic Effect** (electric current along B-field)
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- **Chiral Dipole Wave** (dipole moment induced by B-field)
- The field $\theta(x)$ itself: **Chiral Magnetic Wave** (propagating imbalance between the number of left- and right-handed quarks)



Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos \cos \rangle - \langle \sin \sin \rangle$$

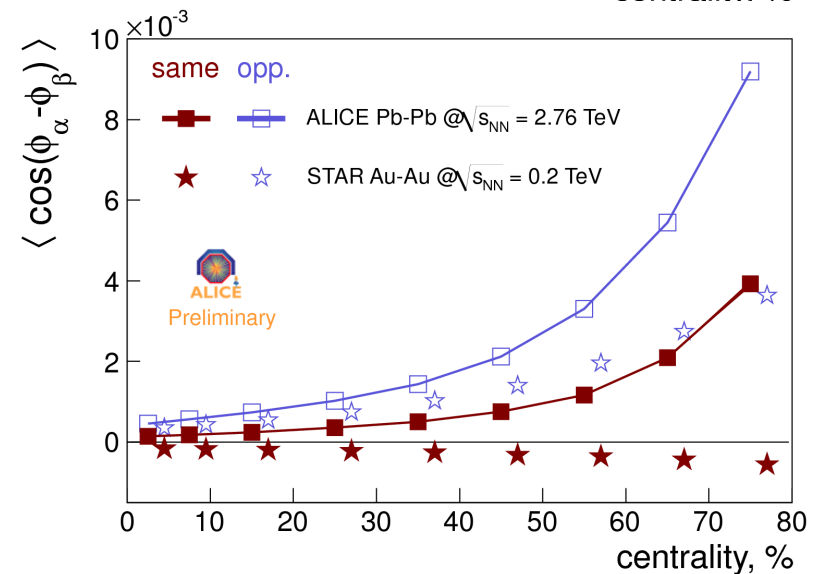
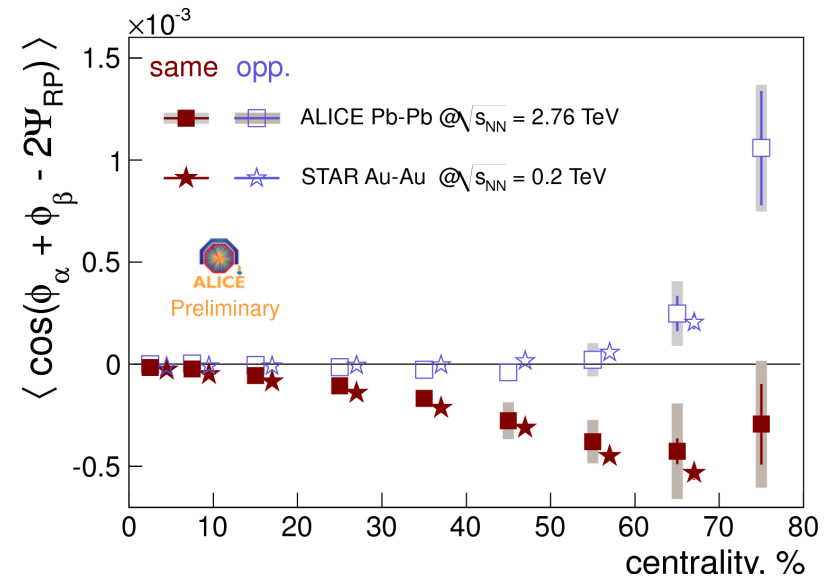
$$\delta_{\alpha,\beta} = \langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos \cos \rangle + \langle \sin \sin \rangle$$



Experiment

$$\gamma_{\alpha,\beta} = \langle \cos(\phi_\alpha + \phi_\beta) \rangle = \underbrace{\langle \cos \cos \rangle}_{\text{in-plane}} - \underbrace{\langle \sin \sin \rangle}_{\text{out-of-plane}}$$

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Experiment

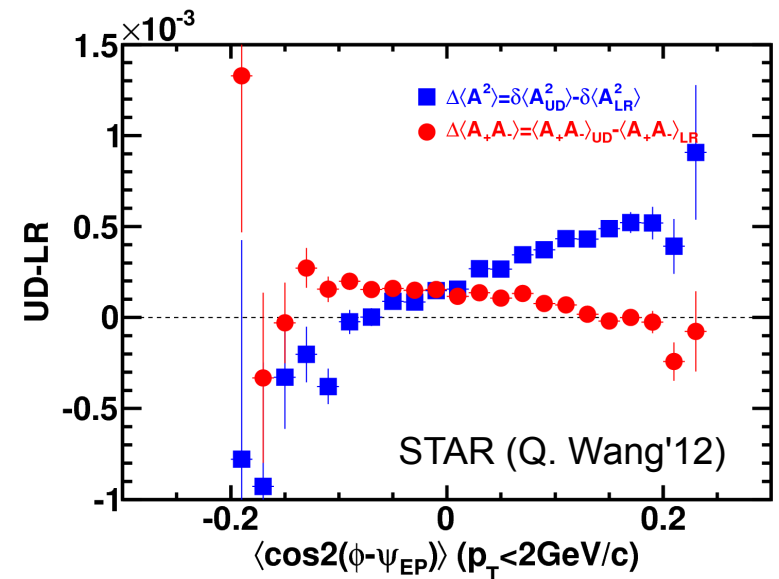
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in-plane
out-of-plane

$$\gamma_{\alpha,\beta} \sim v_2 F_{\alpha,\beta} - H_{\alpha,\beta}^{\text{out}} + H_{\alpha,\beta}^{\text{in}}$$

$$\delta_{\alpha,\beta} \sim F_{\alpha,\beta} + H_{\alpha,\beta}^{\text{out}} + H_{\alpha,\beta}^{\text{in}}$$



Experiment

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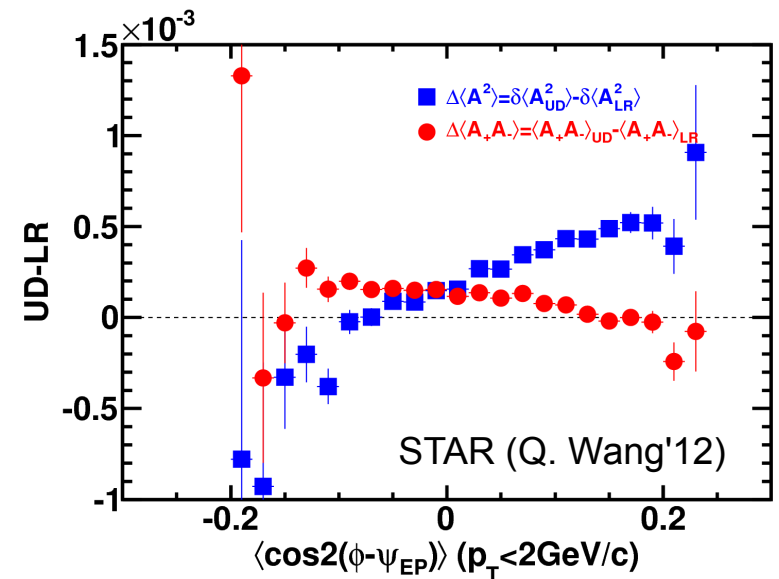
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in-plane out-of-plane

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flow-dependent flow-independent



Experiment

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in-plane out-of-plane

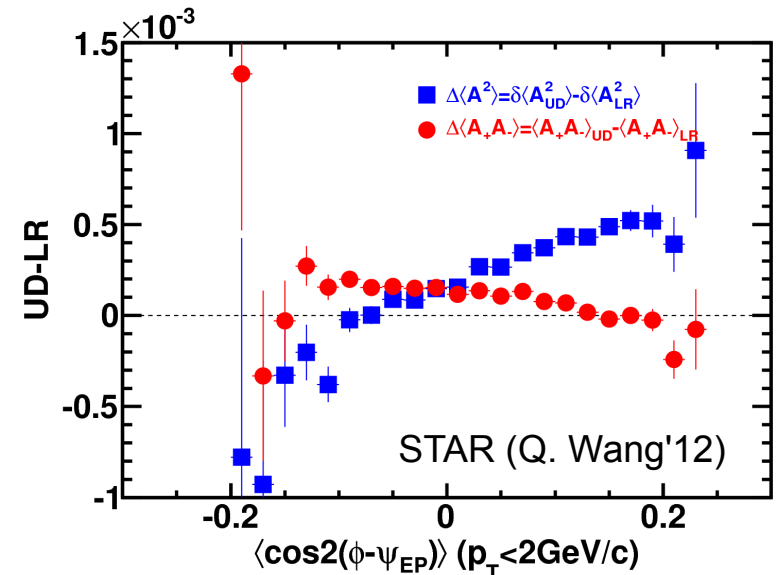
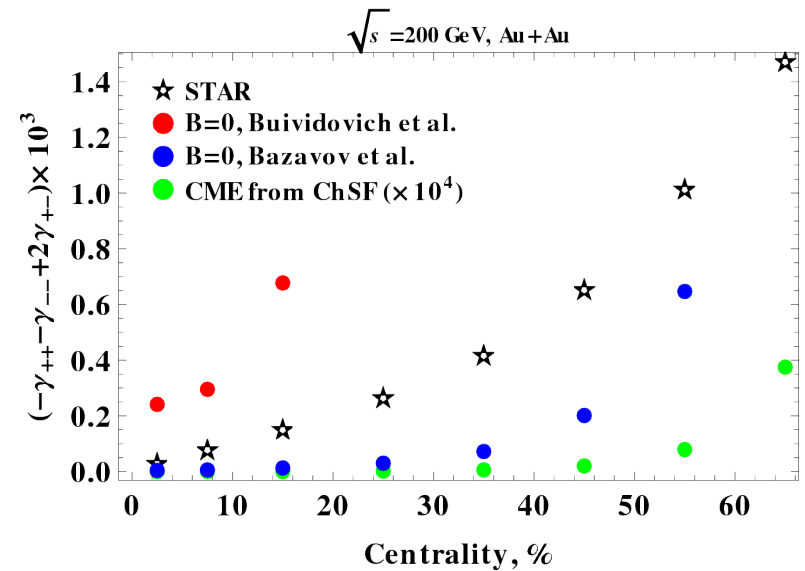
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flow-dependent flow-independent

$$H_{++} + H_{--} - 2H_{+-} \sim \frac{4\pi\tau^2\rho^2\mathcal{R}^2}{3N_q^2} \left(\langle j_{\parallel}^2 \rangle + 2\langle j_{\perp}^2 \rangle \right)$$

Buividovich, Chernodub, Luschevskaya, Polikarpov' 09



Interesting projects

- Add more flavors. The „axion-like“ field might be just a pion. At strong B it will be then a 2D pion.
- Role of the low-dimensional defects in inducing superfluidity/superconductivity. Copenhagen vacuum.
- Entropy and thermalization of various parts of fermionic spectrum. Relation to the vacuum entropy.
- Considering vortices (axionic strings). One might reproduce the chiral vortical effect.
- Holographic model of the chiral superfluidity and high-order corrections.
- Experimental searches for the mentioned effects.
- Chiral electric effect on a lattice.

Thank you for the attention!

and

Have a good time!

**All comments on the papers are welcome!
Also feel free to ask questions about the experimental observables.**

Backup slide

